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Bayes Estimation under Conjugate Prior for the Case of Laplace Double Exponential Distribution

Md. Habibur Rahman[1] and M. K. Roy[2]

[1]*Department of Statistics, Chittagong University, Chittagong-4331, Bangladesh*
[2]*Ranada Prasad Shaha University, Narayanganj, Bangladesh*

Abstract

The Bayesian estimation approach is a non-classical device in the estimation part of statistical inference which is very useful in real world situation. The main objective of this paper is to study the Bayes estimators of the parameter of Laplace double exponential distribution. In Bayesian estimation loss function, prior distribution and posterior distribution are the most important ingredients. In real life we try to minimize the loss and want to know some prior information about the problem to solve it accurately. The well known conjugate priors are considered for finding the Bayes estimator. In our study we have used different symmetric and asymmetric loss functions such as squared error loss function, quadratic loss function, modified linear exponential (MLINEX) loss function and non-linear exponential (NLINEX) loss function. The performance of the obtained estimators for different types of loss functions are then compared among themselves as well as with the classical maximum likelihood estimator (MLE). Mean Square Error (MSE) of the estimators are also computed and presented in graphs.

Keywords: *Squared Error Loss Function; Modified Linear Exponential Loss Function; Non-Linear Exponential Loss Function; Maximum Likelihood Estimator; Bayes Estimator Under Quadratic Loss Function.*

Statistical Inference এ বেইজের প্রাক্কলন একটি নন ক্লাসিক্যাল উপায় যা বাস্তব বিশ্বে খুবই প্রয়োজন। এ গবেষণার মুল উদ্দেশ্য হচ্ছে Laplace double exponential Distribution এর প্যারামিটারের জন্য বেইজের প্রাক্কলক নির্ণয় করা। Loss Function, prior dristribution এবং Posterior dristribution হল বেইজের প্রাক্কলনের গুরুত্বপূর্ণ উপাদান। বাস্তব জীবনে আমরা ক্ষতি কমানোর চেষ্ঠা করি এবং সমস্যার যথার্থ সমাধানে সে সম্পর্কে কিছু prior information জানতে চাই। বেইজের প্রাক্কলক নির্ণয়ে এ গবেষণায় Conjugate prior ব্যবহার করা হয়েছে। বিভিন্ন ধরনের প্রতিসম এবং অপ্রতিসম Loss function যেমন Squared Error loss function, Quadratic loss function, Modified linear exponential (MLINEX) loss function এবং Non Linear exponential (NLIEX) loss function ব্যবহার করা হয়েছে। বিভিন্ন Loss Function এর জন্য অর্জিত প্রাক্কলক সমুহের দক্ষতা নিজেদের মধ্যে এবং Maximum likelihood Estimator (MLE) এর সাথেও তুলনা করা হয়েছে। প্রাক্কলক সমুহের Mean square error (MSE) নির্ণয় করা হয়েছে এবং তা লেখে দেখানো হয়েছে।

1. Introduction

The Laplace distribution is a continuous probability distribution named after Pierre-Simon Laplace. It is also sometimes called the double exponential distribution, because it can be thought of as two exponential distributions (with an additional location parameter) spliced together back-to-back. The Laplace distribution can be used to model various real world problems. The distribution can find most interesting and successful application in modeling of financial data such as to model growth rates, stock prices, annual gross domestic production, interest and forex rates [1]. It has generally two parameters. One is location parameter and other is scale parameter. Here only scale parameter is considered to estimate.

A continuous random variable X is said to have a generalised Laplace double exponential distribution if its p.d.f is given by [2].

$$f(X;\theta,\lambda) = \frac{1}{2\lambda} e^{-\frac{|x-\theta|}{\lambda}} \quad ; -\infty < x < \infty, \lambda > 0, -\infty < \theta < \infty \qquad (1)$$

$$= 0 \qquad ; \text{otherwise}$$

where λ is the scale parameter and θ is the location parameter.

1.1. Prior and Posterior Density Function

For Bayesian estimation we need to specify a prior and posterior distribution for the parameter. We have from equation (1)

$$f(X;\theta,\lambda) = \frac{1}{2\lambda} e^{-\frac{|x-\theta|}{\lambda}}$$

For convenience replacing $\frac{1}{\lambda}$ by p, we get

$$f(x;\theta,p) = \frac{p}{2} e^{-p|x-\theta|} \quad ; p, x > 0 \qquad (2)$$

Consider a Gamma prior for p having pdf

$$g(p) = \frac{\beta^{\alpha}}{\Gamma\alpha} e^{-\beta p} p^{\alpha-1} ; \alpha, \beta, p > 0 \qquad (3)$$

Now the Posterior density function of p for the given random sample X is given by [3]

$$f(p|x) = \frac{\left(\frac{p}{2}\right)^n e^{-p\sum|x-\theta|} e^{-\beta p} \ p^{\alpha-1}}{\int \left(\frac{p}{2}\right)^n e^{-p\sum|x-\theta|} e^{-\beta p} \ p^{\alpha-1} dp}$$

$$= \frac{e^{-p[\sum|x-\theta|+\beta]} \ p^{\alpha+n-1}}{\int e^{-p[\sum|x-\theta|+\beta]} p^{\alpha+n-1} dp}$$

$$\Rightarrow \quad f\,(p|x) = \frac{[\sum|x-\theta|+\beta]^{\alpha+n}}{\Gamma(\alpha+n)} e^{-p[\sum|x-\theta|+\beta]} \;\; p^{\alpha+n-1} \qquad (4)$$

Which implies that $f\ (p|x)\ \sim G(\alpha+n,\ \sum|x-\theta|+\beta)$, since prior and posterior distribution belongs to the same family hence the prior is conjugate prior.

2. Different Estimators of Parameter $\boldsymbol{\lambda}$

In this section Bayes estimators of parameter λ for different loss functions along with maximum likelihood estimator have been determined.

2.1 MLE OF $\boldsymbol{\lambda}$

Let $X = (X_{1,}X_{2,\ldots\ldots,}X_n)$ be a random sample of size *n* drawn from the Laplace double exponential distribution defined in (1). Then the likelihood function of the parameter λ for the random sample *X* is given by [3]

$$\text{L}\,(\lambda|X) = \prod_{i=1}^{n} f\ (x_i|\lambda)$$

$$\Rightarrow \quad \text{L}\,(\lambda|X) = \left(\frac{1}{2\lambda}\right)^n e^{-\sum_{i=1}^{n}\frac{|x_i-\theta|}{\lambda}} \qquad (5)$$

Taking log on both sides of (5) we get

$$\text{Log L}\,(\lambda|X) = \text{-n log}\,(2\lambda)\text{-} \sum_{i=1}^{n}\frac{|x_i-\theta|}{\lambda}$$

$$\Rightarrow \quad \text{Log L}\,(\lambda|X) = \text{-n log2-n log}\,\lambda\text{-}\sum_{i=1}^{n}\frac{|x_i-\theta|}{\lambda}$$

MLE of λ will be the solution of the equation

$$\frac{d\ \log \text{L}\,(\lambda|X)}{d\lambda} = 0$$

$$\Rightarrow \frac{-n}{\lambda} + \frac{\sum|x_i-\theta|}{\lambda^2} = 0$$

$$\Rightarrow \frac{-n\lambda+\sum|x_i-\theta|}{\lambda^2} = 0$$

$$\Rightarrow \quad n\,\lambda = \sum|x_i - \theta|$$

Hence $\hat{\lambda}_{MLE} = \frac{\sum|x_i-\theta|}{n}$, is the MLE of λ where θ known.

2.2. Bayes Estimator of λ for Squared Error Loss Function

Here we have determined Bayes estimator of $\boldsymbol{\lambda}$ for squared error loss function defined as [4]

$$L\,(t\,;\,p) = (t-p)^2 \qquad (6)$$

For squared error loss function Bayes estimator is the mean of posterior density function. From (4) posterior density function is a Gamma distribution with parameter $(\alpha + n)$ and $(\sum|x-\theta| + \beta)$. Hence the mean of posterior density function is $\frac{(\alpha+n)}{(\sum|x-\theta|+\beta)}$. Therefore the Bayes estimator of p is given by

$$\hat{p}_{BSE} = \frac{\alpha+n}{\sum|x-\theta|+\beta}$$

We have $\frac{1}{\lambda} = p$, hence $\hat{\lambda}_{BSE} = \frac{1}{\hat{p}_{BSE}} = \frac{\sum|x-\theta|+\beta}{\alpha+n}$, is the Bayes estimator of parameter λ under SE loss function.

2.3. Bayes Estimator of λ for Quadratic Loss Function

Now suppose the loss function is quadratic, which is defined as [5]

$$L\,(t\,;\,p) = \left(\frac{t-p}{p}\right)^2 \qquad (7)$$

Under quadratic loss function Bayes estimator of p is obtained by solving the following equation

$$\frac{d}{dt}\int\left(\frac{t-p}{p}\right)^2 \ f(p|x)dp = 0$$

$$\Rightarrow\int \frac{2(t-p)}{p^2} \ f(p|x)dp \quad = 0$$

$\Rightarrow$

$$\frac{[\sum|x-\theta|+\beta]^{\alpha+n}}{\Gamma(\alpha+n)} \ t\int_0^\infty e^{-p[\sum_{i=0}^{n}|x_i-\theta|+\beta]} \ p^{\alpha+n-2-1}dp =$$

$$\frac{[\sum|x_i-\theta|+\beta]^{\alpha+n}}{\Gamma(\alpha+n)} \ \int_0^\infty e^{-p[\sum|x_i-\theta|+\beta]} \ p^{\alpha+n-1-1}dp$$

$$\Rightarrow t \ \frac{\Gamma(\alpha+n-2)}{[\sum|x_i-\theta|+\beta]^{\alpha+n-2}} = \frac{\Gamma(\alpha+n-1)}{[\sum|x_i-\theta|+\beta]^{\alpha+n-1}}$$

$$\Rightarrow \quad t \quad = \frac{\Gamma(\alpha+n-1)}{\Gamma(\alpha+n-2)} \ \frac{[\sum|x_i-\theta|+\beta]^{\alpha+n-2}}{[\sum|x_i-\theta|+\beta]^{\alpha+n-1}}$$

$$\Rightarrow \quad t \quad = \frac{(\alpha+n-2)}{[\sum|x_i-\theta|+\beta]}$$

$$\Rightarrow \quad \hat{p}_{BQL} = \frac{(\alpha+n-2)}{[\sum|x_i-\theta|+\beta]}$$

Since we have $\frac{1}{\lambda} = p$, then $\hat{\lambda}_{BQL} = \frac{[\sum|x_i-\theta|+\beta]}{(\alpha+n-2)}$, is the Bayes estimator of parameter λ under quadratic loss function.

2.4. Bayes Estimator of λ for MLINEX Loss Function

Now let us consider the MLINEX loss function defined as **[**5]

$$L\ (\hat{p};p) = w\left[\left(\frac{\hat{p}}{p}\right)^c - c\log\left(\frac{\hat{p}}{p}\right) - 1\right],\ w>0,\ c\neq 0 \qquad (8)$$

For MLINEX loss function Bayes estimator of p is obtained from [5]

$$\hat{p}_{BML} = [E(p^{-c}|x)]^{\frac{-1}{c}} \qquad (9)$$

Here $E\,(p^{-c}) = \int_0^\infty p^{-c} \ f(p|x)$

$$= \frac{[\sum|x_i - \theta| + \beta]^{\alpha+n}}{\Gamma(\alpha+n)} \int_0^\infty e^{-p[\sum|x_i-\theta|+\beta]}\ p^{\alpha+n-c-1}\ dp$$

$$= \frac{[\sum|x_i - \theta| + \beta]^{\alpha+n}}{\Gamma(\alpha+n)} \frac{\Gamma(\alpha+n-c)}{[\sum|x_i-\theta|+\beta]^{\alpha+n-c}}$$

$$\Rightarrow E(p^{-c}) = \frac{\Gamma(\alpha+n-c)}{\Gamma(\alpha+n)}\ [\sum|x_i - \theta| + \beta]^c$$

Therefore from (9) we get $\hat{p}_{BML} = \left[\frac{\Gamma(\alpha+n-c)}{\Gamma(\alpha+n)}\right]^{\frac{-1}{c}} \frac{1}{[\sum|x_i-\theta|+\beta]}$.

We have $\frac{1}{\lambda} = p$, hence $\hat{\lambda}_{BML} = \left[\frac{\Gamma(\alpha+n-c)}{\Gamma(\alpha+n)}\right]^{\frac{1}{c}}\ [\sum|x_i - \theta| + \beta]$, is the Bayes estimator of parameter λ under MLINEX loss function.

2.5 Bayes Estimator of λ for NLINEX Loss Function

Let us consider the following NLINEX loss function of the form [6]

$$L(D) = k\,[\exp(cD) + cD^2 - cD - 1]\ ,\ k > 0\ ,\ c > 0 \qquad (10)$$

where D represents the estimation error.i.e. D = $\hat{p} - p$

For NLINEX loss function Bayes estimator of p is [1]

$$\hat{p}_{BNL} = -\left[lnE_p\{\exp(-cp)\} - 2E_p(p)\right] / (c+2) \qquad (11)$$

where $E_p(.)$ stands for posterior expectation.

Now, $E_p\{\exp(-cp)\} = \int_0^\infty e^{-cp}\quad f(p|x)\,dp$

$$= \frac{[\sum|x_i-\theta|+\beta]^{\alpha+n}}{\Gamma(\alpha+n)} \int_0^\infty e^{-p[c+\sum|x_i-\theta|+\beta\,]}\ p^{\alpha+n-1}\ dp$$

$$= \frac{[\sum|x_i-\theta|+\beta]^{\alpha+n}}{\Gamma(\alpha+n)} \quad \frac{\Gamma(\alpha+n)}{[c+\sum|x_i-\theta|+\beta\,]^{(\alpha+n)}}$$

$$= \frac{[\sum|x_i-\theta|+\beta]^{\alpha+n}}{[c+\sum|x_i-\theta|+\beta\,]^{(\alpha+n)}}$$

$$= \left(1 + \frac{c}{[\sum|x_i-\theta|+\beta]}\right)^{-(\alpha+n)}$$

Hence $\ln E_p\{\exp(-cp)\} = -(\alpha+n)\ln\left(1 + \frac{c}{[\sum|x_i-\theta|+\beta]}\right)$ (12)

Again $E_p(p) = \int_0^\infty p\ f(p\,/\,x)dp$

$$= \frac{[\sum|x_i-\theta|+\beta]^{\alpha+n}}{\Gamma(\alpha+n)} \int_0^\infty e^{-p[\sum|x_i-\theta|+\beta]}\ p^{\alpha+n+1-1}\ dp$$

$$= \frac{[\sum|x_i-\theta|+\beta]^{\alpha+n}}{\Gamma(\alpha+n)} \frac{\Gamma(\alpha+n+1)}{[\sum|x_i-\theta|+\beta]^{\alpha+n+1}}$$

$$\Rightarrow \quad E_p(p) = \frac{(\alpha+n)}{[\sum|x_i-\theta|+\beta]} \tag{13}$$

Using (12) and (13) in (11) we get

$$\hat{p}_{BNL} = -[-(\alpha+n)\ln\left(1+\frac{c}{[\sum|x_i-\theta|+\beta]}\right) - 2\ \frac{(\alpha+n)}{[\sum|x_i-\theta|+\beta]}] \ / \ (c+2)$$

$$\Rightarrow \hat{p}_{BNL} = (\alpha+n)\ [\ln\left(1+\frac{c}{[\sum|x_i-\theta|+\beta]}\right) + \frac{2}{[\sum|x_i-\theta|+\beta]}] \ / \ (c+2)$$

We have $\frac{1}{\lambda} = p$, hence $\hat{\lambda}_{BNL} = \frac{(c+2)}{(\alpha+n)\ [\ln\left(1+\frac{c}{[\sum|x_i-\theta|+\beta]}\right)+\frac{2}{[\sum|x_i-\theta|+\beta]}]}$, is the Bayes

estimator of parameter λ under NLINEX loss function.

3. Empirical Study

To compare the estimators $\hat{\lambda}_{MLE}$, $\hat{\lambda}_{BSE}$, $\hat{\lambda}_{BQL}$, $\hat{\lambda}_{BML}$, and $\hat{\lambda}_{BNL}$we have considered the MSE of the estimators. The MSE of an estimator λ is defined as

$$\text{MSE}(\hat{\lambda}) = \text{E}\left[\ (\lambda - \hat{\lambda})^2\ \right]$$
$$= \text{Var}(\hat{\lambda}) + \left[Bias(\hat{\lambda})\right]^2$$

To obtain the variance of $\hat{\lambda}$, we have used the true value of the parameter λ under consideration. We have obtained the estimated value and MSE of the estimator by using the Monte Carlo simulation method [7] from the Laplace double exponential distribution. Five thousand samples have taken for each case. The results and their graphs are presented bellow.

Table 1. Estimated value and MSE of different estimators of the parameter λ of Laplace double exponential distribution when α =1, β =2, θ = 1, λ =1 and c=1

n	Criteria	$\hat{\lambda}_{MLE}$	$\hat{\lambda}_{BSE}$	$\hat{\lambda}_{BQL}$	$\hat{\lambda}_{BML}$	$\hat{\lambda}_{BNL}$
5	Estimated value	0.6989	0.9150	1.3760	1.0867	0.9386
	MSE	0.2181	0.0917	0.3446	0.1292	0.0890
10	Estimated value	0.6928	0.8118	0.9893	0.8901	0.8275
	MSE	0.1563	0.0860	0.0753	0.0723	0.0816
15	Estimated value	0.6943	0.7716	0.8896	0.8208	0.7826
	MSE	0.1359	0.0874	0.0614	0.0720	0.0817
20	Estimated value	0.6930	0.7601	0.8293	0.7919	0.7633
	MSE	0.1251	0.0859	0.0616	0.0736	0.0840
25	Estimated value	0.6933	0.7442	0.8074	0.7747	0.7485
	MSE	0.1180	0.0881	0.0643	0.0760	0.0854
30	Estimated value	0.6945	0.7371	0.7866	0.7598	0.7445
	MSE	0.1137	0.0888	0.0674	0.0788	0.0843

From the above table it is seen that the MSE of $\hat{\lambda}_{MLE}$ is largest for almost all cases except for sample size 5. On the other hand, among the non-classical estimators $\hat{\lambda}_{BQL}$ shows smallest MSE for different sample size only, it is the highest when sample size is 5 (Figure-1).

Figure 1. Graph of MSEs of different estimates of parameter λ of Laplace double exponential distribution when α=1, β=2, θ=1, λ=1 and c=1

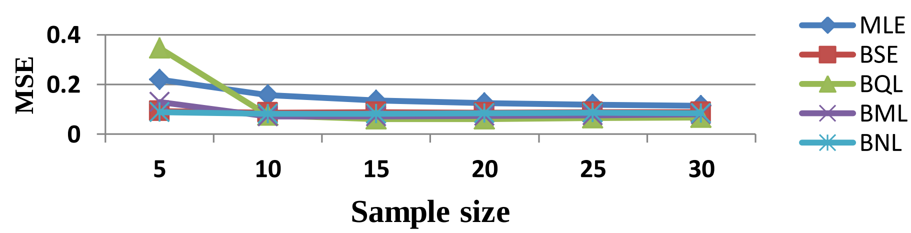


Table 2. Estimated value and MSE of different estimators of the parameter λ of Laplace double exponential distribution when α =1.5, β=2, θ= 1, λ=1.5 and c=1

n	Criteria	$\hat{\lambda}_{MLE}$	$\hat{\lambda}_{BSE}$	$\hat{\lambda}_{BQL}$	$\hat{\lambda}_{BML}$	$\hat{\lambda}_{BNL}$
5	Estimated value	1.0483	1.1130	1.6093	1.3154	1.1402
	MSE	0.4907	0.3660	0.3660	0.2605	0.3029
10	Estimated value	1.0370	1.0776	1.3048	1.1767	1.0880
	MSE	0.3526	0.2839	0.1909	0.2287	0.2724
15	Estimated value	1.0415	1.0679	1.2101	1.1373	1.0641
	MSE	0.3042	0.2670	0.1836	0.2196	0.2633
20	Estimated value	1.0643	1.0624	1.1575	1.1123	1.0723
	MSE	0.2509	0.2523	0.1892	0.2152	0.2437
25	Estimated value	1.0387	1.0554	1.1390	1.1002	1.0625
	MSE	0.2675	0.2461	0.1864	0.2133	0.2383
30	Estimated value	1.0418	1.0575	1.1247	1.0890	1.0597
	MSE	0.2558	0.2384	0.1896	0.2129	0.2355

Table 2 represents largest value of MSE for $\hat{\lambda}_{MLE}$ in all cases. It is also clear from table 2 that MSE of $\hat{\boldsymbol{\lambda}}_{BQL}$is smallest than others estimators for different sample size (figure 2).

Figure 2. Graph of MSEs of different estimates of parameter λ of Laplace double exponential distribution when α=1.5, β=2, θ=1, λ=1.5 and c=1

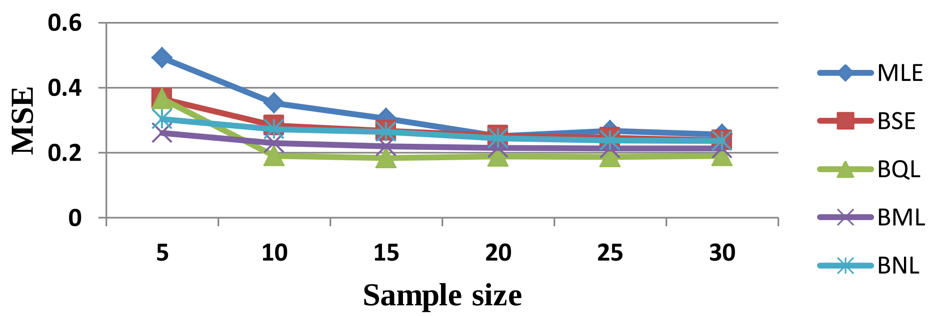


Table 3. Estimated value and MSE of different estimators of the parameter λ of Laplace double exponential distribution when α =0.5, β=1, θ= -1, λ=2 and c=2

n	Criteria	$\hat{\lambda}_{MLE}$	$\hat{\lambda}_{BSE}$	$\hat{\lambda}_{BQL}$	$\hat{\lambda}_{BML}$	$\hat{\lambda}_{BNL}$
5	Estimated value MSE	1.3955 0.8551	1.4312 0.7389	2.2456 1.0067	2.0144 0.8070	1.5344 0.6539
10	Estimated value MSE	1.3755 0.6379	1.4259 0.5558	1.7602 0.3879	1.6564 0.4280	1.4640 0.5118
15	Estimated value MSE	1.3784 0.5448	1.4179 0.4967	1.6143 0.3590	1.5547 0.3907	1.4390 0.4735
20	Estimated value MSE	1.3808 0.5023	1.4032 0.4767	1.5546 0.3433	1.5022 0.3847	1.4290 0.4458
25	Estimated value MSE	1.3861 0.4743	1.3981 0.4561	1.5135 0.3460	1.4873 0.3675	1.4190 0.4289
30	Estimated value MSE	1.3869 0.4558	1.3995 0.4403	1.4951 0.3479	1.4702 0.3662	1.4165 0.4198

Table.3. shows the variation in the performance of the estimator for different sample size. More or less similar pattern are observed here as previous tables that is MSE of $\hat{\boldsymbol{\lambda}}_{\boldsymbol{MLE}}$ is higher than all other estimators. MSE of $\hat{\boldsymbol{\lambda}}_{\boldsymbol{BQL}}$ is least in the class of Bayes estimators for increasing size of samples (Figure 3).

Figure 3. Graph of MSEs of different estimates of parameter λ of Laplace double exponential distribution when α=0.5, β=1, θ=-1, λ=2 and c=2

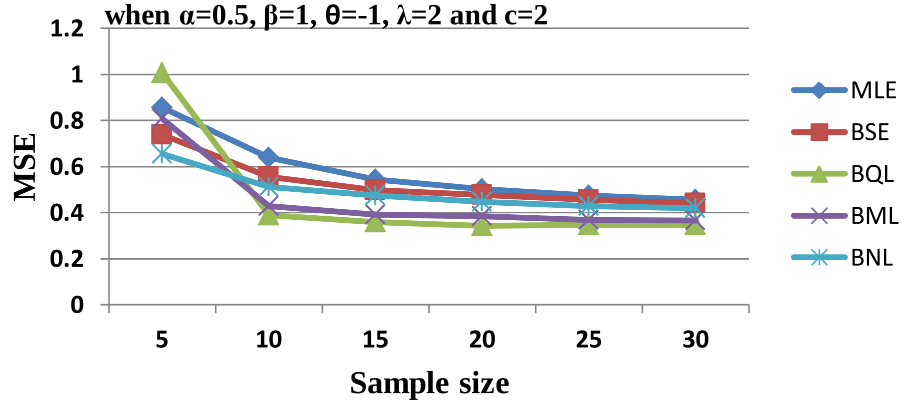


Table 4. Estimated value and MSE of different estimators of the parameter of Laplace double exponential distribution when α =1, β=1, θ= 1, λ=1 and c=1

n	Criteria	$\hat{\lambda}_{MLE}$	$\hat{\lambda}_{BSE}$	$\hat{\lambda}_{BQL}$	$\hat{\lambda}_{BML}$	$\hat{\lambda}_{BNL}$
5	Estimated value	0.6867	0.7491	1.1225	0.9008	0.7714
	MSE	0.2198	0.1514	0.2053	0.1398	0.1377
10	Estimated value	0.6930	0.7223	0.8779	0.7903	0.7349
	MSE	0.1550	0.1288	0.0893	0.1048	0.1216
15	Estimated value	0.6943	0.7134	0.8104	0.7629	0.7180
	MSE	0.1350	0.1195	0.0820	0.0980	0.1142
20	Estimated value	0.6943	0.7067	0.7814	0.7434	0.7142
	MSE	0.1240	0.1126	0.0819	0.0966	0.1091
25	Estimated value	0.6922	0.7094	0.7634	0.7337	0.7134
	MSE	0.1184	0.1081	0.0834	0.0966	0.1050
30	Estimated value	0.6919	0.7048	0.7529	0.7305	0.7100
	MSE	0.1153	0.1063	0.0821	0.0936	0.1032

For different sample size table 4 also shows minimum values of MSE for $\hat{\boldsymbol{\lambda}}_{BQL}$. On the other hand $\hat{\boldsymbol{\lambda}}_{MLE}$ keep its tradition as previous cases (Figure 4).

Figure 4. Graph of MSEs of different estimates of parameter λ of Laplace double exponential distribution when α=1, β=1, θ=1, λ=1 and c=1

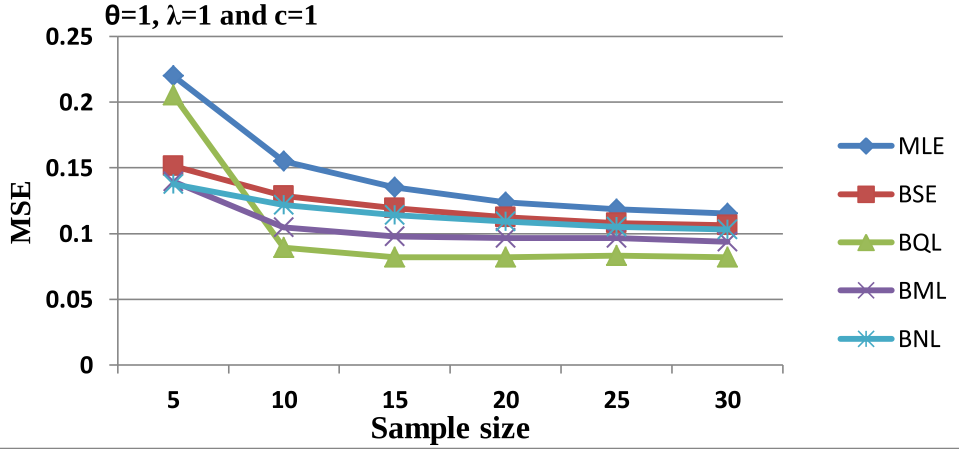
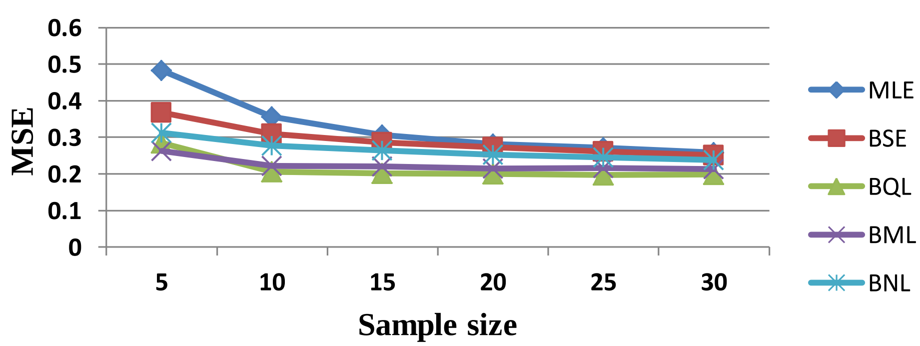


Table 5. Estimated value and MSE of different estimators of the parameter λ of Laplace double exponential distribution when α=2, β=2, θ = -2, λ=1.5 and c=2

n	Criteria	$\hat{\lambda}_{MLE}$	$\hat{\lambda}_{BSE}$	$\hat{\lambda}_{BQL}$	$\hat{\lambda}_{BML}$	$\hat{\lambda}_{BNL}$
5	Estimated value	1.0473	1.0289	1.4422	1.3110	1.0926
	MSE	0.4826	0.3674	0.2846	0.2620	0.3120
10	Estimated value	1.0398	1.0395	1.2363	1.1876	1.0753
	MSE	0.3554	0.3097	0.2055	0.2225	0.2772
15	Estimated value	1.0384	1.0369	1.1676	1.1345	1.0587
	MSE	0.3058	0.2856	0.2020	0.2201	0.2646
20	Estimated value	1.0396	1.0366	1.1369	1.1131	1.0583
	MSE	0.2818	0.2727	0.2005	0.2152	0.2524
25	Estimated value	1.0342	1.0399	1.1250	1.0963	1.0545
	MSE	0.2716	0.2612	0.1968	0.2165	0.2457
30	Estimated value	1.0401	1.0401	1.1073	1.0881	1.0558
	MSE	0.2585	0.2504	0.1991	0.2135	0.2386

Table 5 gives smaller values of MSE for $\hat{\lambda}_{BQL}$ than all other estimators in the study and in some cases it is very near to that of $\hat{\lambda}_{BML}$ (figure 5).

Figure 5. Graph of MSEs of different estimates of parameter λ of Laplace double exponential distribution when α=2, β=2, θ = -2, λ=1.5 and c=2



The performance of the estimators for different values of parameter theta (θ), alpha (α), Beta (β), are also shown in the subsequent tables along with their graphical presentation.

Table 6. Estimated value and MSE of different estimators of the parameter λ of Laplace double exponential distribution for different values of θ and n=10, α =1, β=1, λ =1 and c=1

θ	Criteria	$\hat{\lambda}_{MLE}$	$\hat{\lambda}_{BSE}$	$\hat{\lambda}_{BQL}$	$\hat{\lambda}_{BML}$	$\hat{\lambda}_{BNL}$
-1	Estimated value MSE	0.6909 0.1539	0.7173 0.1330	0.8820 0.0906	0.7901 0.1050	0.7378 0.1202
-1.5	Estimated value MSE	0.6903 0.1560	0.7212 0.1277	0.8809 0.0885	0.7905 0.1027	0.7380 0.1191
-2	Estimated value MSE	0.6953 0.1529	0.7206 0.1265	0.8870 0.0917	0.8009 0.1038	0.7329 0.1238
1	Estimated value MSE	0.6951 0.1558	0.7283 0.1250	0.8825 0.0870	0.7918 0.1040	0.7376 0.1207
1.5	Estimated value MSE	0.6889 0.1546	0.7185 0.1289	0.8806 0.0895	0.7909 0.1061	0.7409 0.1201
2	Estimated value MSE	0.6909 0.1557	0.7185 0.1297	0.8815 0.0902	0.8003 0.1019	0.7354 0.1209

Table 6. also shows smaller MSE of $\hat{\lambda}_{BQL}$ for different values of θ (figure 6).

Figure 6. Graph of MSEs of different estimates of parameter λ of Laplace double exponential distribution when n=10, α=1, β=1, λ=1 and c=1

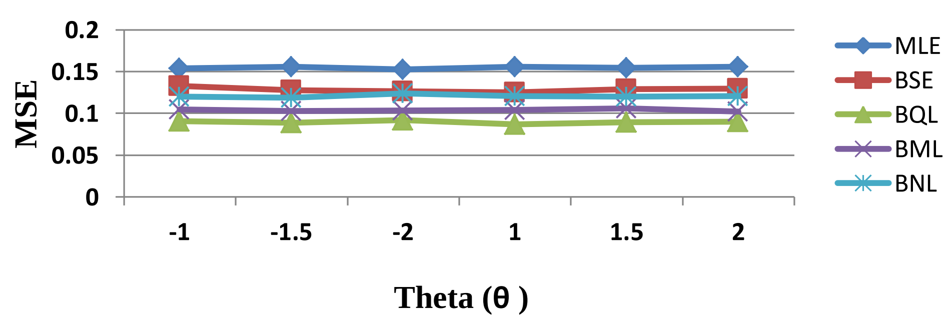


Table 7. Estimated value and MSE of different estimators of the parameter λ of Laplace double exponential distribution for different values of α and n=15, θ=2, β=2, λ=1 .5, c=2

α	Criteria	$\hat{\lambda}_{MLE}$	$\hat{\lambda}_{BSE}$	$\hat{\lambda}_{BQL}$	$\hat{\lambda}_{BML}$	$\hat{\lambda}_{BNL}$
0.5	Estimated value	1.0415	1.1284	1.3084	1.2582	1.1703
	MSE	0.3027	0.2236	0.1524	0.1670	0.1976
1	Estimated value	1.0411	1.1034	1.2530	1.2063	1.1207
	MSE	0.3028	0.2375	0.1637	0.1839	0.2246
1.5	Estimated value	1.0373	1.0633	1.2170	1.1741	1.0920
	MSE	0.3043	0.2636	0.1809	0.1952	0.2442
2	Estimated value	1.0479	1.0322	1.1783	1.1322	1.0656
	MSE	0.2994	0.2886	0.2012	0.2237	0.2623

In table 7. It is seen that as α increases all MSEs show rising trend except that of MLE which is parallel to the horizontal axis. Here MSE of $\hat{\lambda}_{BQL}$ increases at a smaller rate than all other estimators.

Figure 7. Graph of MSEs of different estimates of parameter λ of Laplace double exponential distribution when n=15, β=2, θ =2, λ=1.5 and c=2

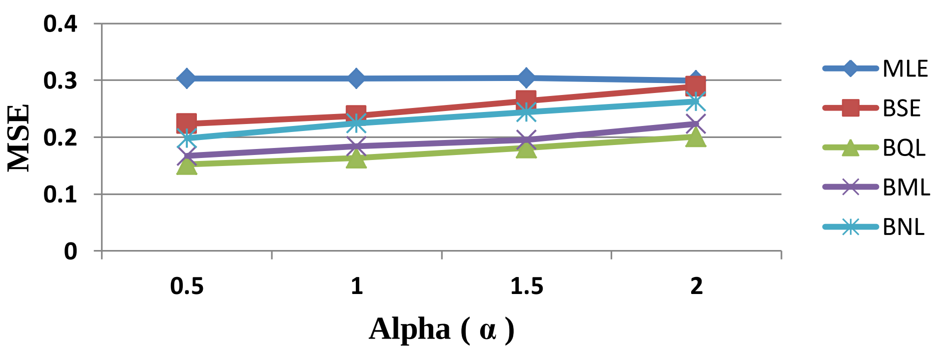


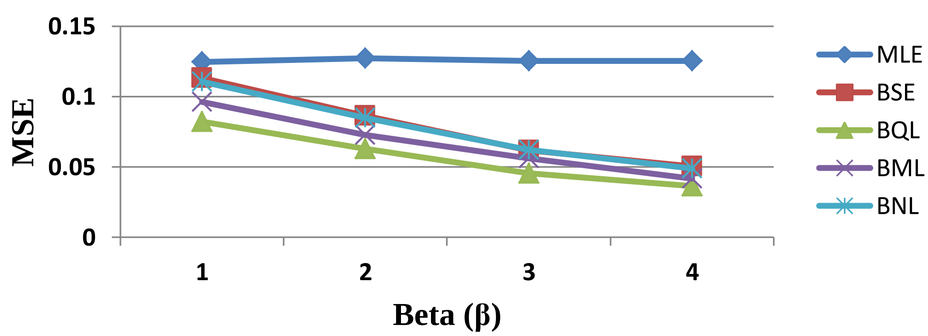
Table 8. Estimated value and MSE of different estimators of the parameter λ of Laplace double exponential distribution for different values of β and n=20, α =1, θ =1, λ=1, c=1

β	Criteria	$\hat{\lambda}_{MLE}$	$\hat{\lambda}_{BSE}$	$\hat{\lambda}_{BQL}$	$\hat{\lambda}_{BML}$	$\hat{\lambda}_{BNL}$
1	Estimated value	`0.6946	0.7081	0.7779	0.7432	0.7120
	MSE	0.1247	0.1131	0.0821	0.0961	0.1106
2	Estimated value	0.6886	0.7579	0.8270	0.7954	0.7607
	MSE	0.1270	0.0863	0.0629	0.0726	0.0848
3	Estimated value	0.6935	0.8047	0.8877	0.8394	0.8132
	MSE	0.1253	0.0617	0.0454	0.0561	0.0623
4	Estimated value	0.6936	0.8492	0.9378	0.8928	0.8550
	MSE	0.1252	0.0504	0.0364	0.0417	0.0487

Table 8. presents declining pattern of MSE for all non-classical estimators as **β** increases.

In the above tables $\hat{\lambda}_{BQL}$ is also showing better performance according to MSE criterion.

Figure 8. Graph of MSEs of different estimates of parameter λ of Laplace double exponential distribution when n=20, α=1, θ=1, λ=1 and c=1



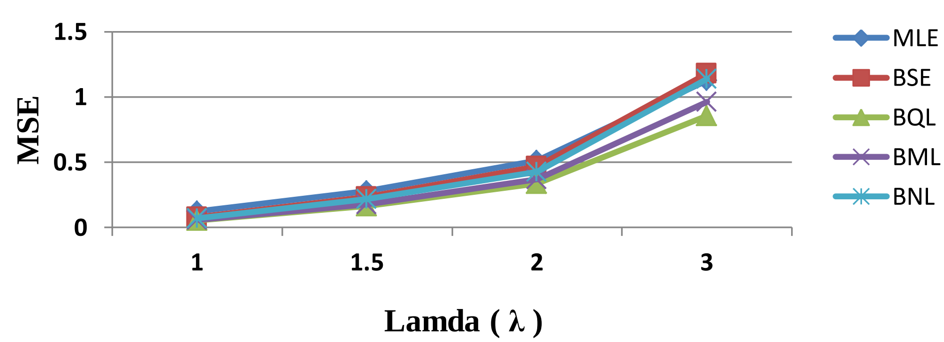
The performance of the estimators is checked in the following table for different values of λ.

Table 9. Estimated value and MSE of different estimators of the parameter λ of Laplace double exponential distribution when n=20, α =2, β =3, θ =2 and c=2

λ	Criteria	$\hat{\lambda}_{MLE}$	$\hat{\lambda}_{BSE}$	$\hat{\lambda}_{BQL}$	$\hat{\lambda}_{BML}$	$\hat{\lambda}_{BNL}$
1	Estimated value	0.6937	0.7657	0.8432	0.8247	0.7903
	MSE	0.1246	0.0802	0.0549	0.0594	0.0694
1.5	Estimated value	1.0454	1.0789	1.1925	1.1629	1.1001
	MSE	0.2788	0.2323	0.1648	0.1758	0.2168
2	Estimated value	1.3773	1.3949	1.5376	1.4971	1.4239
	MSE	0.5127	0.4667	0.3362	0.3696	0.4275
3	Estimated value	2.0816	2.0238	2.2348	2.1585	2.0443
	MSE	1.1264	1.1799	0.8554	0.9628	1.1367

In this case also the MSE of $\hat{\lambda}_{BQL}$ is least among the estimators considered in the study (figure 9).

Figue 9. Graph of MSEs of different estimates of parameter λ of Laplace double exponential distribution when n=20, α=2, β=3, θ =2 and c=2



Conclusions

From the above simulation results analysis and graphical study we can conclude that except for few cases Bayes estimator under quadratic loss is better than other estimators in the study. Thus quadratic loss function can be suggested for the estimation of parameters from Laplace Double Exponential Distribution in case of non-classical method.

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