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## Force Feed Effects on Process Stability in Turning

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### Abstract

This paper presents analytical stability analysis of turning using non-linear force-feed model in two dimensions. Most of the existing analytical models ignored the effect of static feed term on the regeneration phenomenon. In practice there is a marked effect of feed on stability due to force variation. The modified analytical equations for cutting insert using three dimensional tool geometry are obtained by considering relative motion of tool with respect to a two dimensional elastic model of work-piece. The critical stability limits obtained as a function of feed are confirmed with time domain analysis. Experiments are conducted on a flexible work-piece at varying feed conditions. The measured cutting forces show a marked effect of feed on stability. A neural network models developed to obtain the critical depth of cut at various values of operating speeds and feeds.

**Key words:** Dynamics modeling, Tool geometry; Force-feed: Stab.

### Introduction

Prediction of unstable operating conditions is vital important in machining practice. The unstable phenomenon known as chatter eventually leads to a tool damage and loss of work surface. Machine tool operators often select the conservative process parameters from the stability lobe diagrams. Analytical prediction of stability in turning and boring is well established with flexible cutting tool models. One of the earliest models was developed by Merrit (1965). Later on, the mechanics of instability was recognized by Tlustý and Poláček (1963) and Tobias and Fishwick (1958). They observed the cutting forces have been dynamically influenced by the modulated chip thickness. Tlustý and Poláček (1963) model employed only one direction along the chip thickness and it can be still effectively employed in straight turning without inclination angles. Later many authors have shown stability in with two dimensional models (Kaneko *et al.*, 1984; Minis *et al.*, 1990 and Rao and Shin 1999). Ozdagnalar and Endres presented analytical stability analysis of multidimensional cutting systems (Ozdagnalar and Endres 2000). The more general model requires inclusion of oblique cutting conditions and multidimensional dynamics for obtaining accuracy. In this regards more recently Ozlu and Budak illustrated a multidimensional turning model for oblique cutting (Ozlu and Budak 2007). This accounts relative tool motion in both feed and rotation directions and also includes the effects of nose radius on the chatter stability.

In all the above chatter analysis procedures a linear force-model is used to assess the stability. These do not account the

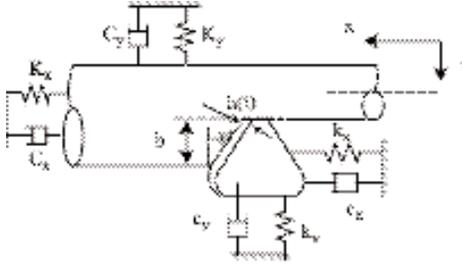
static feed variations in regeneratin models. In practice force various nonlinearly with feed and depth of cut. To study the effect of nonlinear force. Zhang and Ni approximately included the force-feed nonlinearity in by considering modulation phase difference in the process (Zhang and Ni 1995). Landers and Ulsoy demonstrated the methods of handling force-feed nonlinearity in chatter predictions (Landers and Ulsoy 2008). Analysis is developed for turning and milling processes (Insperger *et al.*, 2008). considered such a force-feed terms in their model while explaining the state-dependent regenerative delay concept. A similar force-feed effect taken in boring operations is demonstrated by Pratt and Nafeh (Pratt and Nayfeh 2001). But these non-linear force-feed models are one-dimensional and cannot be used in oblique cutting conditions (Pratt, 2001).

In the present paper analytical multidimensional stability equations are formulated by considering nonlinear force-feed effect in turning using three-dimensional tool geometry. Feed and radial forces are included in the model appropriately and dynamic equations are developed. The revised expression for stable depth of cut as a function chatter frequency is derived. Effect of feed on stability shown in stability lobe diagrams have been tested in time domain by solving the coupled delay differential equations. Based on the critical depth of cut at various conditions of feed and speed, experiments are conducted on a flexible work-piece to conform the effect of feed on stability.

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**Dynamic cutting force models**

The basic parameters in turning process are chip thickness  $h$ , the depth of cut  $b$  and the cutting angles. The side cutting edge angle ( $\Psi$ ) is measured on the rake face of the tool. The relation between the forces and chip thickness is obtained by considering the coordinate systems:  $x$  (feed),  $y$  (radial) and  $z$  (cutting) as shown in Fig. 1. Dynamic displacement in cutting ( $z$ ) direction does not affect the chip thickness.



**Fig.1 Dynamic Model of turning**

( $b$ : depth of cut,  $h(t)$ : chip-thickness,  $\psi$ : side cutting edge angle)

The modulated chip thickness resulting from vibrations of the tool and work-piece is expressed as :

$$h(t)=h_0\cos\psi+[(x_c(t)-x_w(t-\tau))-(x_c(t-\tau)-x_w(t-\tau))]\cos\psi-[y_c(t)-y_w(t)-(y_c(t-\tau)-y_w(t-\tau))]\sin\psi \tag{1}$$

Here  $h_0$  is the nominal feed per revolution,  $x_c(t)$ ,  $y_c(t)$ ,  $x_w(t)$  and  $y_w(t)$  are the cutter and work-piece displacements in the current pass and  $x_c(t-\tau)$ ,  $y_c(t-\tau)$ ,  $x_w(t-\tau)$ ,  $y_w(t-\tau)$  are that in previous pass along  $x$  and  $y$  directions respectively. Also  $\tau$  is the time required for one spindle revolution.

For convenience, Eq(1) can be written as

$$h(t)=h_s+\Delta x\cos-\Delta y \sin \psi \tag{2}$$

$$\text{where } h_s=h_0\cos \psi \tag{3}$$

$$\Delta x=(x_c(t)-x_w(t))-(x_c(t-\tau) (x_w(t-\tau)) \tag{4}$$

$$\Delta y=(y_c(t)-y_w(t))-(y_c(t-\tau)y_w(t-\tau)) \tag{5}$$

The cutting operation is three dimensional in nature due to the existence of inclination angle. Now the forces at the cutting edge are modeled with oblique cutting model. The forces acting on the cutting edge are feed force  $F_f$  and radial force  $F_r$  which are expressed as

$$F_f = \frac{K_f}{\cos\psi} bh^a(t) \tag{6}$$

$$F_r = \frac{K_r}{\cos\psi} bh^\beta(t) \tag{7}$$

Here  $K_f$ ,  $a$ ,  $K_r$ ,  $\beta$  are the empirically determined cutting coefficients in feed and radial directions. Substituting  $h(t)$

from Eq.(2) and linerizing the forces about  $h_s$ , we can write Eqs. (6) and (7) as

$$F_f = \frac{K_f}{\cos\psi} b [h_s^a + a h_s^{a-1} (\Delta x \cos\psi - \Delta y \sin \psi)] \tag{8}$$

$$F_r = \frac{K_r}{\cos\psi} b [h_s^\beta + \beta h_s^{\beta-1} (\Delta x \cos\psi - \Delta y \sin \psi)] \tag{9}$$

Since static chip-thickness does not contribute to the regeneration, the first term in equations (8) and (9) can be eliminated as in earlier models.

Now resolving the forces  $F_f$  and  $F_r$  along  $x$  and  $y$  directions:

$$F_x = F_f\cos\psi + F_r \sin\psi = \frac{b}{\cos\psi} [(p\Delta y + q\Delta x) \sin\psi \cos\psi - (p\Delta x \cos^2\psi + q\Delta y \sin^2\psi)] \tag{10}$$

$$F_y = F_f\sin\psi + F_r \cos\psi = \frac{b}{\cos\psi} [(p\Delta y - q\Delta x) \sin\psi \cos\psi - (p\Delta y \cos^2\psi - q\Delta x \sin^2\psi)] \tag{11}$$

where  $p=K_f \alpha h_s^{a-1}$  and  $q=K_r \beta h_s^{\beta-1}$

The equations can be written in matrix form as

$$\begin{Bmatrix} F_x \\ F_y \end{Bmatrix} = \frac{1}{\cos\psi} \begin{bmatrix} -\cos\psi & \sin\psi \\ \sin\psi & \cos\psi \end{bmatrix} \begin{Bmatrix} K_f \alpha b h_s^{a-1} \\ K_r \beta h_s^{\beta-1} \end{Bmatrix} \begin{bmatrix} \cos\psi & -\sin\psi \\ \sin\psi & \cos\psi \end{bmatrix} \begin{Bmatrix} \Delta x \\ \Delta y \end{Bmatrix} \tag{12}$$

These equations can be rewritten as

$$\{F\}=b\{A\} \{\Delta d\} \tag{13}$$

$$\text{where } (A) = \frac{1}{\cos\psi} \begin{bmatrix} -\cos\psi & \sin\psi \\ \sin\psi & \cos\psi \end{bmatrix} \begin{Bmatrix} K_f \alpha b h_s^{a-1} \\ K_r \beta h_s^{\beta-1} \end{Bmatrix} \begin{bmatrix} \cos\psi & -\sin\psi \\ \sin\psi & \cos\psi \end{bmatrix} \tag{14}$$

**Stability analysis**

Taking Laplace transforms on both sides of Eq. (13)

$$F(s)=b \{A\} (1-e^{-s\tau}) D(s) \tag{15}$$

By considering the transfer function matrix  $\{G(s)\}$  of the system, it is possible to write  $D(s)$  as  $\{G(s)\} F(s)$ . Hence Eq. (15) becomes:

$$F(s)=b\{A\}(1-e^{-s\tau}) [G(s)]f(s) \tag{16}$$

Substituting  $s=j\omega$ , Eq. (16) in frequency domain turns to:

$$F(j\omega s)=b(A) (1- e^{-j\omega\tau}) [G(j\omega)] F(j\omega) \tag{17}$$

This equation has non-trivial solution if and only if its determinant is zero, yielding:

$$\det \{I\} - \lambda[G_0]\}=0 \tag{18}$$

$$\text{where } [G_0]=[A][G(j\omega) \text{ and } \lambda= b(1- e^{-j\omega\tau}) \tag{19}$$

This eigenvalue problem given by Eq. (18) can be directly solved to obtain the complex eigenvalue  $\lambda$  By considering

the transfer matrix as  $[G(j\omega)]$

$$= \begin{bmatrix} G_{xx}(j\omega) & 0 \\ 0 & G_{yy}(j\omega) \end{bmatrix}, \text{ the Eigen value can be obtained as}$$

$$\lambda = \frac{\cos\psi}{(p\cos^2\psi - q\sin\psi\cos\psi)G_{xx} + (q\sin^2\psi + q\sin\psi\cos\psi)G_{yy}} \quad (20)$$

writing  $\lambda$  from Eq. (19) as

$$\lambda = \lambda_R + \lambda_I = b(1 - \cos \omega\tau) + j b \sin \omega\tau \quad (21)$$

The phase shift  $k = \frac{\lambda_I}{\lambda_R}$  can be defined from Eq. 21 according to

$$\frac{\lambda_I}{\lambda_R} = \frac{\sin\omega\tau}{1 - \cos\omega\tau} = 1/\tan \frac{\omega\tau}{2} = \tan\left(\frac{\pi}{2} + n\pi - \frac{\omega\tau}{2}\right) \quad (22)$$

This gives the critical values of  $\tau$  as

$$\tau^* = \frac{2}{\omega} \left\{ \left(n + \frac{1}{2}\right)\tau - a \tan\left[\frac{\lambda_I}{\lambda_R}\right] \right\} \text{ where } n = 0, 1, 2, \dots \quad (23a)$$

The corresponding critical values of  $b$  is given by

$$b^* = \frac{\lambda_R}{1 - \cos} \quad (23b)$$

By writing spindle speed in rpm as  $N = 60/\tau$ , stability lobe diagram can be plotted between the values of  $b^*$  and  $N$  for different values of  $n$ . The second order delay differential equation in time domain are solved at a specific testing depth of cut and feed conditions using revised fourth order Runge-Kutta method.

**Results and Discussion**

Based on the proposed analytical model, stability analysis is conducted. The cutting material parameters used in the simulation (Ozlu and Budak 2007) are given in Table I.

**Table I. Cutting and material data**

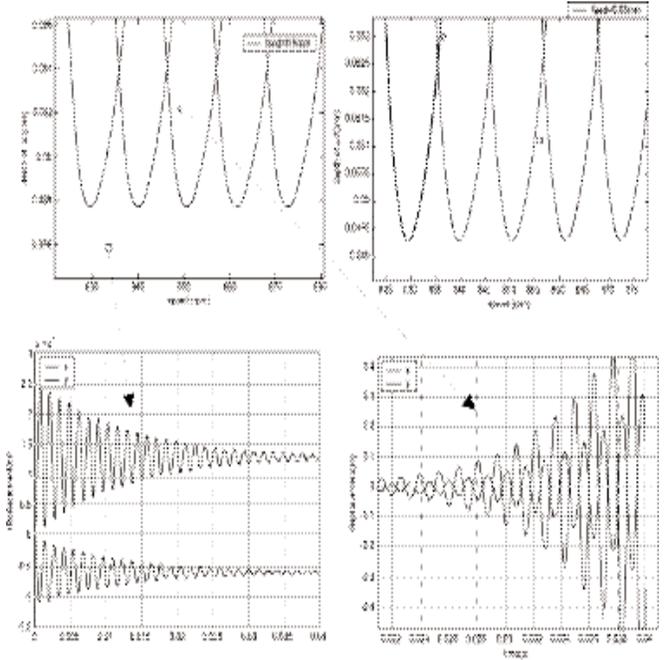
Cutting tool	work-piece
Natural frequency $\omega_{nxc} = \omega_{nyc} = 1100\text{Hz}$	Modulus of elasticity $E = 2.1 \times 10^7 \text{ N/m}^2$
Stiffness $k_x = k_y = 1.2 \times 10^7 \text{ N/m}^2$	Density $\rho = 7800 \text{ Kg/m}^3$
Damping ratio $\zeta_x = \zeta_y = 0.015$	Diameter $d = 39 \text{ mm}$
Cutting coefficients $K_f = 800 \text{ MPa}$	Length $L = 75 \text{ mm}$
$K_r = 128 \text{ MPa}$	
Side cutting edge angle $\psi = 10^\circ$	Average damping ratio $\zeta_{xw} = \zeta_{yw} = 0.025$
Normal rake angle $= 5^\circ$	
Inclination angle $5^\circ$	
Nose radius $= 0.4 \text{ mm}$	
Working feed $= 0.01 - 0.6 \text{ mm}$	

The axial (x) and bending (y) stiffness and corresponding natural frequencies ( $\omega_{nxc}, \omega_{nyc}$ ) of the work are computed by considering cantilever boundary conditions. The tool work transfer function matrices are evaluated according to:

$$G(s) = \begin{bmatrix} \frac{m_{nxc}^2}{k_x (s^2 + 2\zeta_{nxc}\omega_{nxc}s + \omega_{nxc}^2)} & 0 \\ 0 & \frac{m_{nyc}^2}{k_y (s^2 + 2\zeta_{nyc}\omega_{nyc}s + \omega_{nyc}^2)} \end{bmatrix}$$

$$G_w(s) = \begin{bmatrix} \frac{m_{nxc}^2}{k_x (s^2 + 2\zeta_{nxc}\omega_{nxc}s + \omega_{nxc}^2)} & 0 \\ 0 & \frac{m_{nyc}^2}{k_y (s^2 + 2\zeta_{nyc}\omega_{nyc}s + \omega_{nyc}^2)} \end{bmatrix}$$

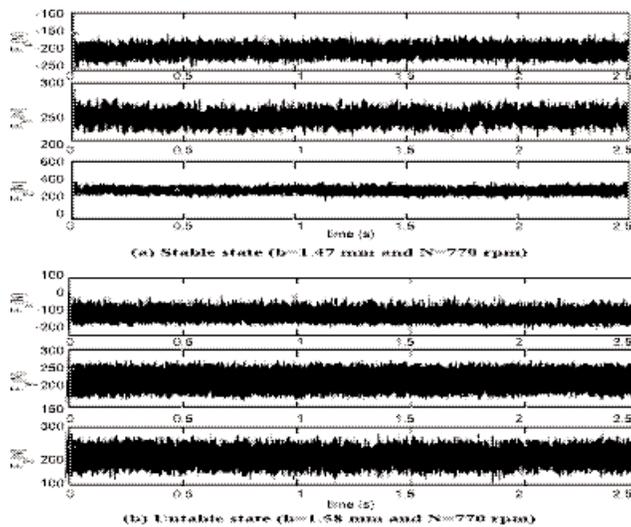
Using the proposed stability model the feed effect on the cutting stability is shown in Fig. 2 as a feed depth stability lobe diagram at speed of  $N_s = 9042 \text{ rpm}$ .



**Fig. 2. Stability lobe diagrams from the proposed model**

Critical depth of cut increases with feed at all the spindle speed values. At low values of a comparatively more change in lobe diagram is noticed. Also it can be seen that the width of lobe has increased due this nonlinear force effect. The effect is verified from the time domain analysis. Fig. 3 shows the time domain response of tool displacement and cutting forces under two different feed conditions at common depth of cut.

Prediction of chatter with time domain analysis requires testing over all the operating regions. Thus it becomes



**Fig. 3. Time domain response of tool displacement and cutting forces**

lengthy procedure in regular cases. On the other hand, the method of frequency sweeping to obtain the stability lobes leads to modeling errors in the process. Hence in order to predict the critical depths of cuts at various operating feeds and speeds, a neural network approach is proposed.

### Conclusion

Effects of nonlinear force-feed conditions in chatter stability of turning operation have been presented in this paper. It is observed that in all the cases the critical depth of cut increase with feed. When the nonlinear feed exponent is less than one as in regular metals, the effect is more pronounced. Practically an increase in feed minimizes the cycle time, but machining forces also increase with feed. So it cannot be used to suppress chatter. Due to the inclusion of feed in chatter relations, it is possible to calibrate the force models at any feed. In practical conditions, the cutting coefficients also change with feed. The influence is ignored in the analytical model.

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