

Mixed convection heat transfer from a heated circular cylinder

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Abstract

This paper performs the effects of thermal buoyancy and the triangular arrangement of circular cylinders on fluid flow and heat transfer within a horizontal channel, the governing equations involving continuity; momentum and energy are solved in two-dimensional, laminar and steady flow regime. The average Nusselt number and drag coefficient are computed for the range of these conditions: $Ri = 0$ to 2 at fixed value of $Pr = 1$, Reynolds number $Re = 30$ and geometrical configurations (blockage ratio of $\beta = 0.1$). In order to observe the flow structure and temperature field under the gradual effect of thermal buoyancy, the streamlines and isotherm contours are illustrated. It is found that, a gradual increase in the value of buoyancy strength creates an asymmetrical flow around the cylinders. Interesting variations of drag coefficient and average Nusselt number are plotted with respect to Richardson number for each cylinder.

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Introduction

The heat transfers from circular cylinders have been vastly examined in previous literature due to its importance in many different field of applied engineering. The study of this kind of research can be the main factor reported for the design, development and maintenance of cooling electronic system, heat exchangers, evaporators, cooling of water and/or oil for a big ships, thermal plants, and radiators of automobiles and so on. Physically, the heat transfer flows form a solid body to fluid domain on three methods, by natural convection, forced convection and mixed convection.

The mixed convection occurs when natural and forced convection mechanisms act simultaneously to transfer heat. The physical parameter that controls the relative influence of convection and forced convection is the Richardson number, it is defined as, $Ri = Gr/Re^2$ with Gr and Re being the Grashof and Reynolds numbers. For $Ri > 1$, the natural convection dominates over the forced convection and for $Ri < 1$ the forced convection dominates. Both the natural and forced convection are equally important when Ri is close to the value 1.

The mixed convection heat transfer from one cylinder or a pair of cylinders for different configurations is the main subject of recent researches. Laidoudi and bouzit (2017) simulated the mixed convection around a pair of confined circular cylinders in a tandem arrangement. The effect of distance between the cylinders, the Reynolds number and the buoyancy strength were the main parameters studied in this paper. Salcedo *et al.* (2016) introduced in this time the effect of unsteady flow regime on mixed convection. Huang *et al.* (2013) also studied in unsteady flow regime the mixed convection around two isothermal and identical square cylinders arranged in tandem and confined channel for moderate values of Reynolds number. Chatterjee and Sakir (2010) performed the effect of thermal buoyancy on fluid flow and heat transfer around two identical square cylinders. Laidoudi *et al.* (2017) investigated the influence of mixed convection on heat transfer around three confined circular cylinders placed in tandem arrangement. Fornarelli *et al.* (2016) tested the effect of aiding and opposing thermal buoyancy on vertical configuration of flow around six circular cylinders placed in-line arrangement.

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The study of mixed convection heat transfer over one cylinder is studied very well in recent years to understand the flow behavior and heat transportation around the cylinder. Chatterjee *et al.* (2013) analyzed through numerical simulations the phenomena of suppression of flow separation around a square and circular cylinder by using positive effect of thermal buoyancy. Laidoudi and Bouzit (2017) tested the effect of aiding thermal buoyancy on upward flow of power-law fluids around a confined circular cylinder.

The fluid flow and mixed convection heat transfer around a three cylinders placed in triangular arrangement is less attention in literature, the first work is that performed by Barros *et al.* (2017) it was a numerical investigation in order to test the effect of aiding buoyancy on the fluid flow and heat transfer over a three circular cylinders placed in triangular arrangement.

Based on the above works, the purpose of this paper is to release through a numerical investigation the study of mixed convection heat transfer and fluid flow around a three circular cylinders placed in triangular arrangement under the radial effect of thermal buoyancy. The distance between the cylinders is fixed. The governing equations are solved at fixed value of Reynolds number of $Re = 30$ and within the range of Richardson number of $Ri = 0$ to 2 and $Pr = 1$.

Problem statement and governing equations

The geometrical configuration under consideration is shown schematically in Figure 1. It is considered three isothermal

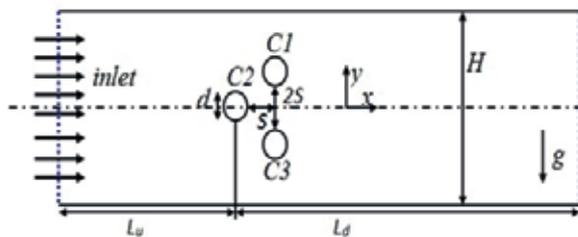


Fig. 1. Schematic diagram of computational domain

and equal circular cylinders, that are located in the cross middle of a long two-dimensional horizontal channel at triangular arrangement. The paper aims to study numerically the incompressible flow of Newtonian fluid around these obstacles. Due to numerical considerations, the fluid flow comes in the channel with uniform velocity profile and constant temperature (T_{in}), it passes the cylinders, whose surfaces are maintained at constant temperature (T_w). The

distance between cylinders (C1 and C3) equals $2S$ and $S = 2d$. The diameter of cylinder is denoted by (d), the ratio of this parameter to the height of the channel, (H), defines the blockage ratio ($\beta = d/H = 0.1$), The upstream distance which is defined as the distance between the center of the cylinder (C2) and the channel inlet (L_w) is 15 times of the cylinder diameter, the downstream distance between the center of the cylinder (C2) and the channel outlet (L_d) is 25 times of the cylinder diameter. The flow and heat transfer phenomena are governed by continuity, momentum and energy equations. This research is based on the following assumptions: laminar incompressible and Newtonian fluid flow, steady regime, mixed convection and constant fluid properties. The

- Continuity

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{1}$$

- Momentum

$$\frac{\partial uu}{\partial x} + \frac{\partial uv}{\partial y} = -\frac{\partial p}{\partial x} + \frac{1}{Re} \left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right) \tag{2a}$$

$$\frac{\partial uv}{\partial x} + \frac{\partial vv}{\partial y} = -\frac{\partial p}{\partial y} + \frac{1}{Re} \left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} \right) + Ri\theta \tag{2b}$$

- Energy

$$\frac{\partial u\theta}{\partial x} + \frac{\partial v\theta}{\partial y} = \frac{1}{PrRe} \left(\frac{\partial^2 \theta}{\partial x^2} + \frac{\partial^2 \theta}{\partial y^2} \right) \tag{3}$$

governing equations subjected to the Boussinesq approximation and negligible dissipation effects are written in their dimensionless forms as follows:

where u , and v are the dimensionless velocity components along x - and y -directions of a Cartesian Coordinate system, respectively, P is the dimensionless pressure and $Re = (u_{max} d/\eta)$ is the Reynolds number based on the cylinder dimension. $Ri = (Gr/Re^2)$ is the Richardson number.

$Gr = g\beta (T_w - T_{in})d^3/\eta^2$ is the Grashof number with g and β being the gravitational acceleration and volumetric expansion

$$u = \frac{\bar{u}}{u_{max}}, v = \frac{\bar{v}}{u_{max}}, x = \frac{\bar{x}}{d}, y = \frac{\bar{y}}{d}, p = \frac{\bar{p}}{\rho u_{max}^2}, \theta = \frac{(T - T_{in})}{T_w - T_{in}} \tag{4}$$

coefficient, θ is the dimensionless temperature and $Pr = \eta/\alpha$ is the Prandtl number. The fluid properties are described by the density ρ , kinematic viscosity η and thermal diffusivity α .

$$u = 1, v = 0, \theta = 0 \quad (5)$$

The dimensionless variables are defined as:
The boundary conditions used in this study are as follows:

$$u = 0, v = 0, \theta = 1 \quad (6)$$

- At the inlet an uniform flow of incompressible fluid with a constant temperature is described

$$u = 0, v = 0, \text{Adiabatic} \quad (7)$$

- On surfaces of obstacle cylinders: The standard no-slip condition is used for flow condition and the cylinders are

$$\frac{\partial u}{\partial x} = 0, \frac{\partial v}{\partial x} = 0, \frac{\partial \theta}{\partial x} = 0 \quad (8)$$

maintained and heated with a constant temperature T_w .

$$C_D = \frac{2F_D}{\rho u_{\max}^2 d} \quad (9)$$

- At the upper and down walls of channel, the usual no-slip condition for flow and adiabatic condition for energy are used.

$$Nu_l = \frac{hd}{k} = -\frac{\partial \theta}{\partial n_s} \quad (10)$$

At the outlet Neumann boundary condition for field variables is employed:

The drag coefficient is calculated from:

$$Nu = \frac{1}{S} \int_s Nu_l ds \quad (11)$$

F_D is the total drag force on the surface of the cylinder. The distribution of local Nusselt number on each surface of cylinder was evaluated for constant wall temperature as:

where h and n_s are: the local surface heat transfer coefficient and the normal direction to the cylinder surface. These local values on entire surface were then averaged to obtain the average Nusselt number of circular cylinder.

Numerical methodology

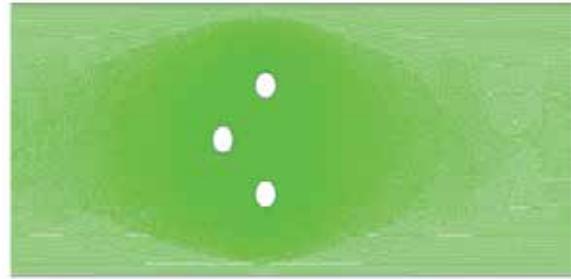


Fig. 2. Typical grids used for simulation

The conservation equations subjected to the aforementioned boundary conditions are solved using a finite volume based CFD solver ANSYS-CFX version (14.0). The numerical methodology used for this paper is similar with that used in our previous work Laidoudi and

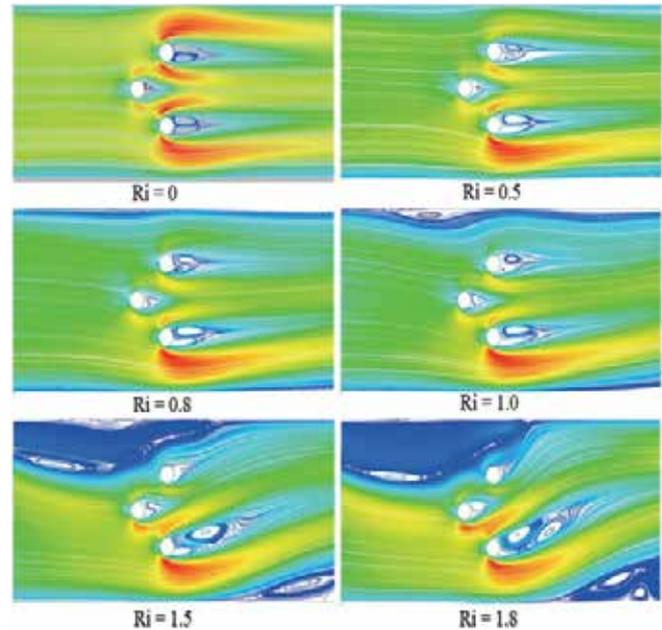


Fig. 3. Streamlines contours around the cylinders for $Ri = 0$ to 1.8 , $Pr = 1$ and $Re = 30$

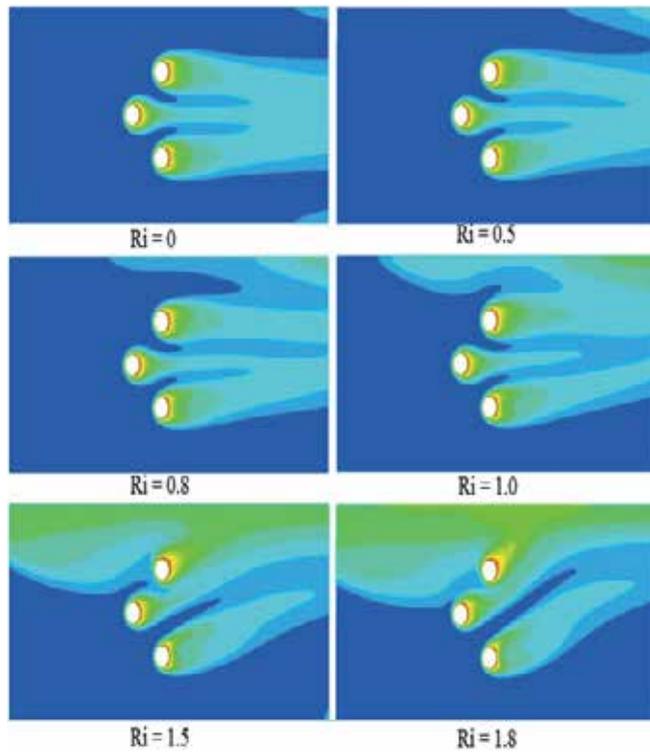


Fig. 4. isotherm contours around cylinders for $Ri = 0$ to 1.8 , $Pr = 1$ and $Re = 30$

Bouzit (2017). Fig. 2 shows the grid point distributions around the cylinders.

Result and discussion

This paper presents numerical simulations of mixed convection heat transfer from a three circular cylinders placed in triangular arrangement for some range of conditions. The representative streamline and isotherm contours are illustrated around the three cylinders for different values of Richardson number as it is shows respectively in the fig. 3 and 4. Fig. 3 shows that the flow patterns are perfectly symmetric with respect to the line ($x = 0$) for the value $Ri = 0$ due to absence of thermal buoyancy. For this value of Richardson number, the recirculation zone sizes behind the upper and lower cylinders (C1 and C3) are greater than recirculation size of the middle cylinder (C2). The gradual increase in the value of Richardson number from 0 to 1.8 loses the symmetry of flow structure around the cylinders; this is due to the thermal buoyancy strength which increases progressively with the gradual increase in the value of Richardson number. The buoyancy creates a radial

velocity of fluid particles close to the cylinders and the particles move toward the upper channel wall. Consequently, the fluid velocity below the cylinders increases owing to the mass conservation principle. Accordingly, the flow structure becomes asymmetric. The degree of asymmetric is seen to be increased with gradual increase in thermal buoyancy strength. Additionally, interesting variations of recirculation zone is seen with respect to Richardson number, increase in Richardson number suppresses the recirculation zones behind the upper and middle cylinders (C1 and C2), meanwhile, the length zones increases behind the lower cylinder (C3). Moreover, it is also observed that, the buoyancy strength up to the value 1 of Richardson number creates some counter rotating regions over the upper cylinder (C1) and lower cylinder (C3).

As mentioned above, figure 4 shows the isotherms around the cylinders placed in triangular arrangement for different values of Richardson number from 0 to 1.8 at fixed value of Reynolds number $Re = 30$ and $Pr = 1$. The isotherms are the reflection of streamlines. It means that the isotherms are symmetric at the value 0 of Richardson number and the asymmetrical distribution of isotherms appears up to the value $Ri = 0$ and it increases gradually with the gradual increase in the value of Richardson number. It is also observed that the isotherms are crowded on surface the front part of cylinders indicating higher value of heat transfer rate and consequently higher value

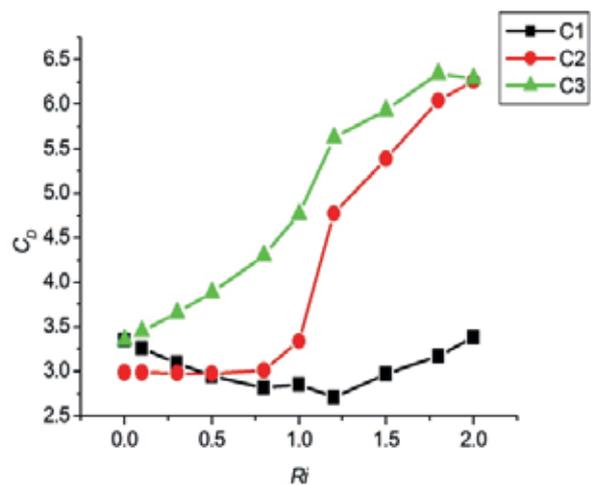


Fig. 5. Variation of total drag coefficient with Richardson number for three cylinders numbers at $Re = 30$ and $Pr = 1$

of Nusselt number. The isotherm crowding around the first cylinder (C1) is observed to be reduced with gradual increase in Richardson number. Meanwhile, it is observed to be increased around the middle and lower cylinders (C2 and C3) due to increase of velocity in the lower part of channel and consequently increases the evacuation of heat transfer rate.

Fig. 5 shows the variation of total drag coefficient with Richardson number at fixed Reynolds and Prandtl numbers for the three cylinders. It is observed that drag coefficient decreases with increase in Richardson number for the upper cylinder up to the value 1.2 and it increases again. Meanwhile, the drag coefficient of the middle and lower cylinders increases with corresponding increase in Richardson number. This is due to the fact that with an increase in Ri , the value of static pressure decreases in the zone behind the cylinders (C2 and C3) which increases the pressure differences between the front and rear parts of cylinder.

Fig. 6 shows the variation of average Nusselt number versus Richardson number at fixed value of Reynolds and Prandtl numbers. As it is understood from the visualization of isotherms and streamlines, the velocity of fluid decreases around the first cylinder with increase in

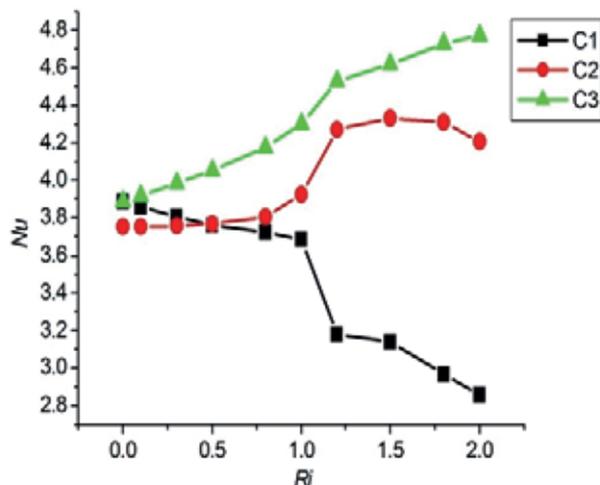


Fig. 6. Variation of average Nusselt number with Richardson number for three cylinders numbers at $Re = 30$ and $Pr = 1$

Richardson number, meanwhile it increases around the second and third cylinders progressively which gradual increase in Richardson number which progressively increases the evacuation of heat transfer rate. So the plotted graphs substantiate this idea, the average Nusselt number decreases with an increase in Ri for the upper cylinder but it increases with Ri for the middle and lower cylinders.

Conclusion

The numerical simulation is carried out to study the mixed convection heat transfer and fluid flow around a three circular cylinders placed in triangular arrangement. The two-dimensional steady laminar flow regime is considered for this work. The computational results are presented and discussed for the range of these conditions $Ri = 0$ to 2, at $Re = 30$ and $Pr = 1$ and fixed geometrical configuration. It is found that the recirculation zone behind the upper cylinder is seen to be reduced with increase in Richardson number meanwhile it increases for the middle and lower cylinders. The total drag coefficient of upper cylinder shows a negative dependence on Richardson number, in the contrary, the total drag coefficient of middle and lower cylinders increases gradually with increase Richardson number. The average Nusselt number decreases with increase in Richardson number for the upper cylinder but it increases for the middle and lower cylinders with increasing Richardson number.

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