

MONTE CARLO SIMULATION OF NEUTRON AND GAMMA RAY BEHAVIOR IN THE PGAA SYSTEM OF THE TRIGA REACTOR USING THE PHITS CODE

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Abstract

This study presents a detailed simulation of the neutron and photon flux distribution as well as their energy spectra within the PGAA system of the Moroccan TRIGA Mark II reactor. The simulation was carried out using the PHITS Monte Carlo code. Several configurations were analyzed, particularly the collimator, the sapphire filter, the closed shutter, and the open shutter. The simulation results show that the sapphire filter significantly attenuates gamma rays, thereby reducing contamination of the PGAA detector. The closed shutter configuration effectively blocks neutrons and photons, especially fast neutrons, while the open shutter configuration allows the complete passage of the neutron beam with well-collimated propagation, thus providing valuable information on the performance of the PGAA system.

Keywords: *Neutron-gamma, Monte Carlo, PHIT, Sapphire, Collimator, Closed shutter, Open shutter*

1. INTRODUCTION

As part of the advancement of non-destructive neutron analysis techniques, a Prompt Gamma Activation Analysis (PGAA) system was installed in the Moroccan TRIGA Mark II research reactor at the National Center for Energy, Sciences, and Nuclear Techniques (CNESTEN), in partnership with the International Atomic Energy Agency (IAEA) [1]. This pool type reactor operates at a power of 2 MW and uses low enriched uranium moderated by graphite. It is equipped with a natural convection cooling system and a graphite reflector. The PGAA system was installed at the end of the tangential NB1 channel (see Figure 1) [2], a channel specifically designed to enhance the extraction of thermal neutrons while significantly attenuating the direct flux of fast neutrons and gamma radiation [3]. This channel passes through the biological shielding of the TRIGA reactor, mainly composed of heavy concrete and steel, and opens into the experimental hall, where the PGAA system is operational. It is a non-destructive nuclear analytical technique, widely recognized for its high sensitivity and accuracy [2]. This method is based on the irradiation of a target with a neutron

beam, which activates the atoms present in the target and causes them to emit prompt gamma radiation at characteristic energies [4].

These gamma photons are then detected and analyzed through a steel collimator using a detection system, typically a scintillation detector or a high-purity germanium (HPGe) detector, installed outside the shielding of the TRIGA reactor [5].

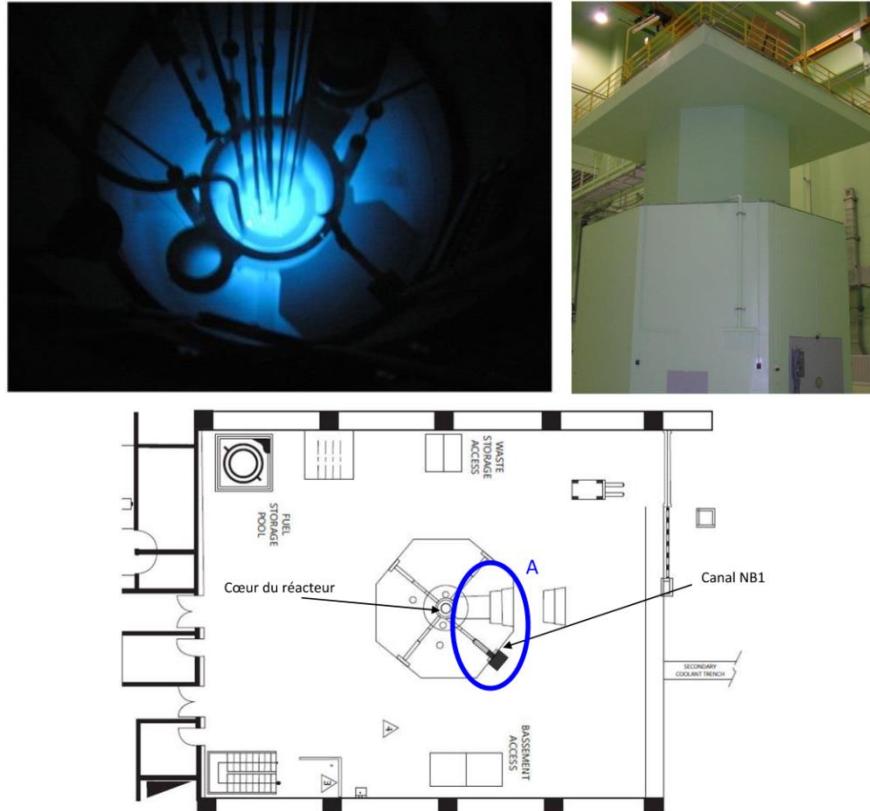


Fig.1. Overall presentation of the TRIGA reactor enclosure and the experimental sector

The installation of the PGAA technique focuses on several essential components, namely: a collimator that precisely directs and filters neutron beams while eliminating divergent gamma photons, a sapphire (Al_2O_3) filter, a two-position beam shutter to control the primary beam (open and closed), a supermirror neutron guide, a beam shaper, a high-purity germanium (HPGe) detector, and a beam stop device [1-2]. Each component plays an important role in ensuring the optimal performance and accuracy of the PGAA analysis by providing the necessary neutron activation and the precise detection of the emitted gamma photons.

This article focuses on the simulation of the PGAA setup for the TRIGA MARK II reactor, primarily based on calculating the flux distribution and the spectrum of gamma photons produced during

neutron interactions with target samples. The aim is to assess the effectiveness of the PGAA technique in a specific experimental configuration and to optimize its application for various nuclear analyses. To achieve this, the PHITS code, developed by the Japan Atomic Energy Agency [4] and based on the Monte Carlo method, was used to examine the impact of different experimental configurations, particularly the steel collimator, the open shutter, and the closed shutter [6-7].

2. MATERIALS AND METHODS

Version 3.34 of the PHITS code (Particle and Heavy Ion Transport Code System) was used to perform the Monte Carlo simulations in this article. PHITS is a versatile system that enables the transport of particles such as photons, neutrons, and ions through various complex geometries [8]. In this article, PHITS was employed to simulate the distribution of neutron and gamma-ray flux, as well as their energy spectra, within the PGAA system [8]. The simulations were carried out using the neutron source spectrum characteristic of the Moroccan TRIGA MARK II reactor [7]. The simulation includes a steel collimator, a sapphire (Al_2O_3) filter, and the collimator-shutter assembly in both open and closed positions [3].

The system simulation incorporates the geometry, shape, and dimensions of each PGAA component. The main element is the neutron collimator, designed to efficiently guide cold or thermal neutrons. It consists of an aluminum (Al) tube ensuring internal vacuum and tightness, along with ten carbon steel rings of grade E235 [9]. These rings have outer diameters ranging from 19.8 cm to 19.95 cm, with central holes reduced from 6.8 cm to 5.6 cm to allow the neutron beam to pass through, and are assembled along a total length of 100 cm [2]. A sapphire filter (Al_2O_3), 5 cm in diameter and 5 cm in length, is positioned at $z = 95$ cm to attenuate the fast neutron component of the beam [10]. This type of filter is particularly effective at allowing gamma photons to pass while reducing neutron intensity.

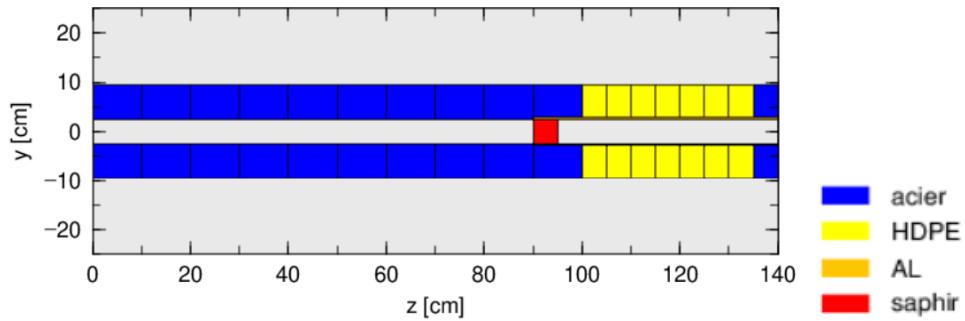


Fig. 2. An overview of the PGAA setup at TRIGA MARK II, including the collimator equipped with a sapphire filter

The collimator also includes seven high-density borated polyethylene (B4C-HDPE) rings containing 5% boron, with a 6.6 cm aperture and an outer diameter of 19 cm, distributed along a length of 35 cm [2]. These rings are capable of thermalizing fast neutrons and absorbing slow neutrons. The closure is ensured by an end ring made of carbon steel, treated with the same anti-corrosion coating as the rings of the first steel collimator block [6].

The collimator has an inner radius of 3.41 cm at the entrance and 2.5 cm at the end near the beam shutter, for a total length of 140 cm [2]. The configuration of the collimator components, as simulated using the Monte Carlo PHITS code, is illustrated in Fig. 2.

The primary shutter consists of two distinct parts: the first section incorporates specific absorbing materials designed to completely block the neutron or gamma ray beam passing through the sample (Fig. 3), while the second section (Fig. 4) features a precisely shaped aperture that minimizes neutron leakage by absorbing the remaining divergent neutrons [4].

In summary, the beam shutter measures 50 cm in length and is composed of high-density materials of varying thicknesses, specifically borated polyethylene (Mirrobor), lead, standard polyethylene, steel, and high-density polyethylene (HDPE) [1].

To attenuate high-energy gamma photons emitted from the reactor core, the beam shutter uses a bismuth (Bi) filter positioned at the entrance of its opening, mounted on an aluminum (Al) support for cooling [6]. Overall, in a PGAA system used in a TRIGA reactor, the bismuth filter measures 5 cm in length and 5 cm in diameter, and plays a crucial role in enhancing the detection of gamma photons emitted by the target while reducing interferences from neutrons and background gamma radiation [2].

Thermal neutrons from the TRIGA MARK II reactor were modeled as a radiation source in PHITS [11] represented by a cylindrical surface with a radius of 8 cm and a height of 2.2 cm [2]. To obtain the results, the t-track tally was used to map the neutron-gamma flux distribution on a given surface, while the t-cross tally was employed to analyze the energy spectrum [12].

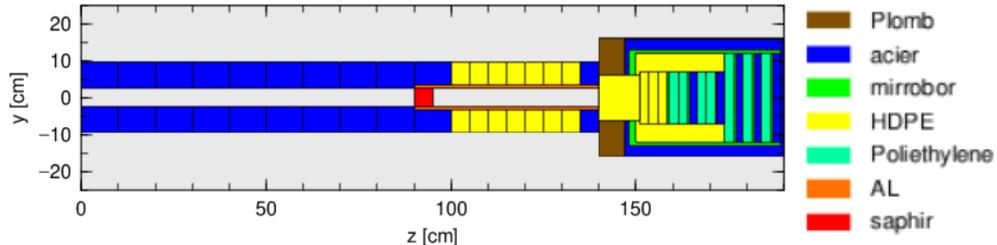


Figure 3. Overview of the PGAA installation at TRIGA MARK II, highlighting the collimator with the shutter in the closed position

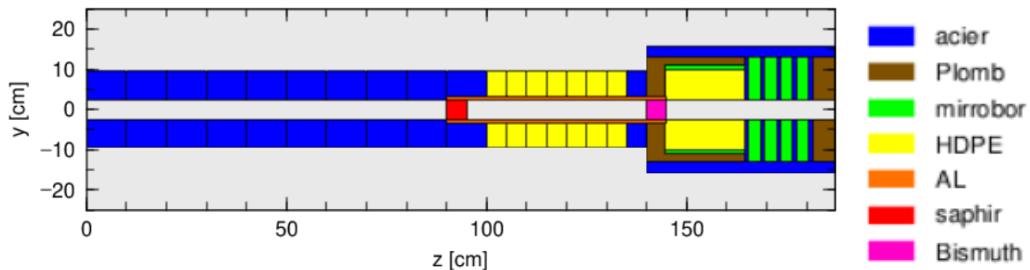


Fig. 4. Overview of the TRIGA MARK II reactors PGAA system, showing the collimator when the beam shutter is in the open position.

In this study, the neutron and Gamma spectra were simulated in the collimator and shutter regions, without the presence of a sample, in order to analyze the secondary gamma fluxes produced by neutron interactions with the surrounding materials of the PGAA (lead, steel, sapphire, HDPE, and polyethylene...) [3]. The observed photons mainly result from neutrons emitted by the reactor core and their interactions with these components. No primary gamma source was explicitly modeled, as the objective was to focus on the secondary gamma rays generated through these interactions [6]. Therefore, the simulated photon spectra in these regions reflect the contribution of prompt gammas induced by neutron–matter interactions, without accounting for the reactor’s primary gamma emissions [1-3].

In this article, several assumptions were made to simplify the simulation process and make the calculations more efficient. First, the initial neutron spectrum emitted by the TRIGA reactor was used as the input source. A fixed irradiation time was assumed for all simulations. Moreover, the geometry of the PGAA system was modeled in an idealized manner to reduce the complexity of the simulation while preserving the physical relevance of the results. All these data were obtained from reactor specifications and validated sources from the literature [1], [13].

To ensure result accuracy, a particle count per batch of 10^6 and a batch number of 10 were selected. The fluence of neutrons and gamma rays was measured using a geometric mesh in various cells, with the collimator serving as the entry zone. Neutron and Photon energies were evaluated from 10^{-9} to 20 MeV with an energy mesh of 70 points to generate a detailed simulation of prompt gamma rays [12,14].

3. RESULTS AND DISCUSSION

The PHITS code was employed to simulate the flux distribution, and the energy spectra of neutrons and gamma rays, (refer to Figs. 5 to 10) [12]. These simulations were conducted on a computer featuring an Intel Core i5 processor (2.30 GHz, 2401 MHz) with 2 cores and 4 threads, operating on Windows 10. Under this configuration, each simulation required approximately 24 hours to complete.

Figures 5 to 7 illustrate the neutron-gamma flux distribution [$1/cm^2/source$] along the PGAA system for three configurations: the collimator alone, the collimator with a closed shutter, and the collimator with an open shutter. Figures 5a and 5b respectively show the spatial distribution of neutron and gamma flux within the collimator, represented in the (y, z) plane. The neutron source is located at $z = 0\text{ cm}$, while the sapphire filter, designed to eliminate unwanted radiation, is positioned at $z = 95\text{ cm}$. In Figure 5a, a high neutron flux intensity is observed at the entrance of the collimator, in the immediate vicinity of the source, around $10^{-3}\text{ [1/cm}^2/source]$. The flux remains concentrated along the central axis of the guide ($y \approx 0\text{ cm}$) and undergoes a gradual attenuation as it approaches the sapphire (Al_2O_3) filter. Beyond the filter ($z > 95\text{ cm}$), the flux is significantly reduced to around $10^{-4}\text{ [1/cm}^2/source]$ along the z-axis, confirming the filter’s effectiveness in partially suppressing fast neutrons. Figure 5b reveals a more diffuse gamma flux distribution. Two notable photon emission zones are visible: the first is located near the entrance ($z \approx 10\text{--}20\text{ cm}$), with an intensity on

the order of 10^{-3} [$1/cm^2/source$], corresponding to initial neutron-matter interactions within the collimator, the second is around $z \approx 95$ cm, coinciding with the position of the sapphire filter. This second zone suggests secondary photon production induced by neutron interactions with sapphire (Al_2O_3), a dense material capable of generating gamma radiation through neutron capture reactions. The gamma flux is generally more spatially spread than the neutron flux, reflecting the greater ability of photons to propagate laterally.

These results confirm the proper functioning of the PGAA system's collimator in its ability to channel useful neutrons while controlling the generation and propagation of secondary radiation. However, special attention should be given to the filter zone, where interactions may alter the overall radiative spectrum.

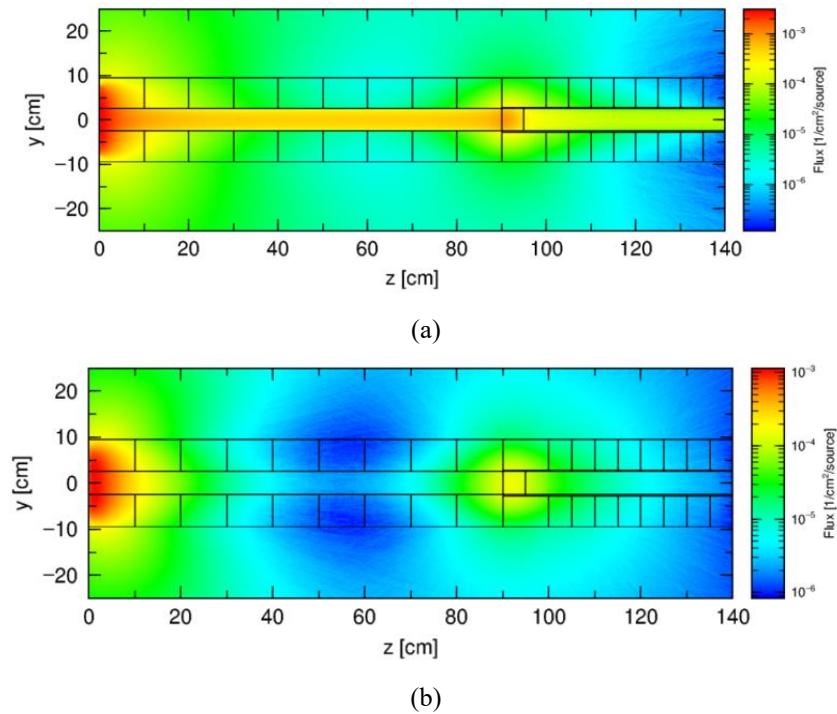


Fig. 5. Neutron- gamma flux at the collimator: (a) Neutron flux at the collimator (b) Gamma flux at the collimator

Figures 6a and 6b respectively illustrate the spatial distribution of the neutron and gamma fluxes within the collimator when the shutter is closed, represented in the (y, z) plane. The introduction of the closed shutter acts as an additional barrier designed to block radiation before it exits the PGAA system. In Figure 6a, a significant attenuation of the neutron flux is observed in the shutter region. The flux, which reaches approximately 10^{-3} [$1/cm^2/source$], at the entrance of the collimator ($z \approx 0$ cm), remains concentrated along the central axis up to the level of the filter. Beyond $z \approx 95$ cm, the flux gradually decreases, before dropping sharply to around 10^{-6} [$1/cm^2/source$], in the closed shutter region. This highlights the highly effective attenuation of neutrons, particularly fast neutrons,

by the closed shutter. Figure 6b shows the distribution of the gamma flux, which, although more spatially extended, also exhibits a clear reduction in the closed shutter region. At the shutter location, the gamma flux drops to approximately 10^{-6} [1/cm²/source], confirming the effectiveness of the device in reducing secondary radiation.

The combined analysis of these distributions highlights the performance of the closed shutter in blocking both neutrons and photons, ensuring enhanced safety outside of irradiation periods. The observed spatial behavior aligns with the design goals of the PGAA system, ensuring both radiological and nuclear protection and effective control of residual radiation.

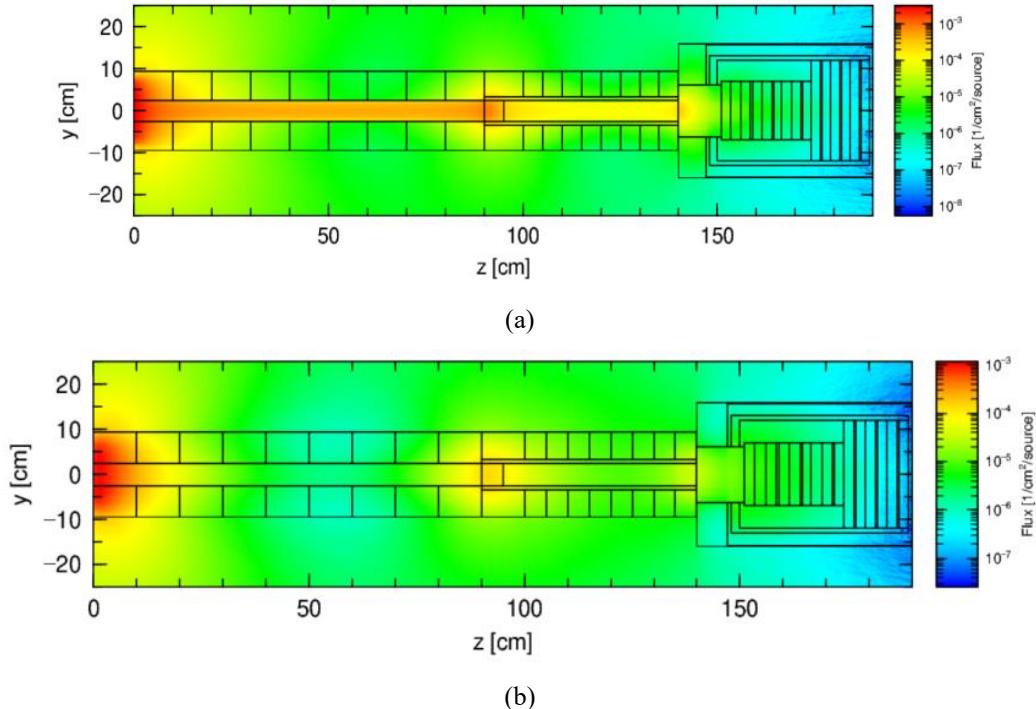


Fig. 6. Neutron- gamma flux at the collimator with closed shutter: (a) Neutron flux at the collimator with closed shutter (b) gamma flux at the collimator with closed shutter

Figures 7a and 7b respectively illustrate the spatial distribution of neutron and gamma flux within the collimator when the shutter is open, represented in the (y, z) plane. This configuration corresponds to the nominal operating state of the PGAA system, during which the neutron beam is allowed to reach the sample in the irradiation chamber. In Figure 7a, the neutron flux retains its maximum intensity at the entrance of the collimator, concentrated along the central axis ($y \approx 0$ cm). Unlike the closed-shutter configuration, no sharp drop is observed in the shutter region, confirming that the shutter is indeed open. Figure 7b shows a more diffuse gamma flux distribution. In this configuration, the gamma flux is fully transmitted to the end of the PGAA system, as expected when the shutter is open, allowing the complete transmission of both the beam and secondary radiation.

Overall, the results confirm that opening the shutter restores the full beam path without significantly altering the neutron and photon flux distributions. The spatial behavior of neutrons and photons is similar to that observed with the steel collimator alone, demonstrating the reliability of the PGAA system in irradiation mode.

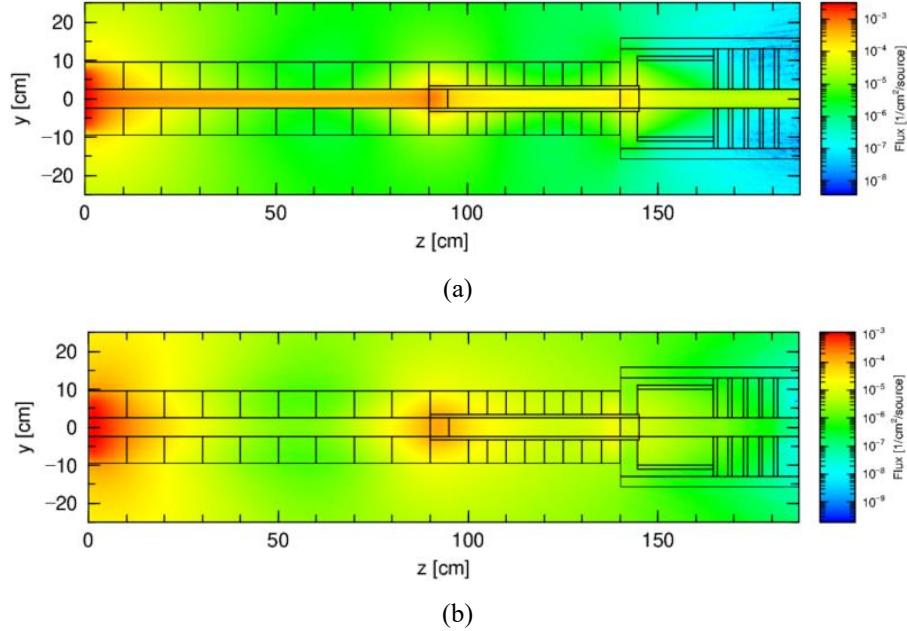


Fig. 7. Neutron- gamma flux at the collimator with an open shutter: (a) Neutron flux at the collimator with an open shutter, (b) Gamma flux at the collimator with an open shutter

The comparison of configurations with the shutter open and closed demonstrated the system's effectiveness in adjusting the neutron flux according to experimental needs (Table 1). Materials such as HDPE and sapphire contribute to moderating neutrons and eliminating parasitic radiation, ensuring optimal neutron flux concentration at the centre and effective reduction of parasitic flux in peripheral areas.

Table1. Comparison of flux in the PGAA system

PGAA system configuration	Central Flux [1/cm ² /source]	Péphérique Flux [1/cm ² /source]
Collimator	$\approx 10^{-4}$	$<10^{-5}$
Closed shutter	$\approx 10^{-6}$	$\approx 10^{-8}$
Opened shutter	$\approx 10^{-4}$	$<10^{-7}$

Figures 8 to 10 show the neutron-photon flux as a function of neutron energy on a logarithmic scale. The neutron flux dominates at low energies, covering a range from thermal neutrons (10^{-9} MeV) to fast neutrons (10 MeV). From 10^{-3} MeV onwards, the photon flux becomes significant and reaches notable intensities up to 10 MeV.

Collimator

The neutron and photon fluxes show in Figure 8 a characteristic distribution as a function of energy. In the thermal neutron range (energy < 0.1 MeV), the neutron flux is high, reaching about 10^6 [1/cm²/MeV/source] confirming the expected dominance of moderated neutrons in this configuration, which is suitable for a collimator designed to slow down neutrons. In contrast, for fast neutrons (energy > 1 MeV), the flux decreases sharply to around 10^{-3} [1/cm²/MeV/source] indicating significant attenuation. Regarding photons, their flux increases significantly starting from 10^{-3} MeV, reaching a peak around 10^{-1} MeV with an intensity of about 4.2483×10^{-1} [1/cm²/MeV/source]. Beyond 1 MeV, the photon flux decreases slightly, reaching a value of 6.4258×10^{-6} [1/cm²/MeV/source] at 10 MeV.

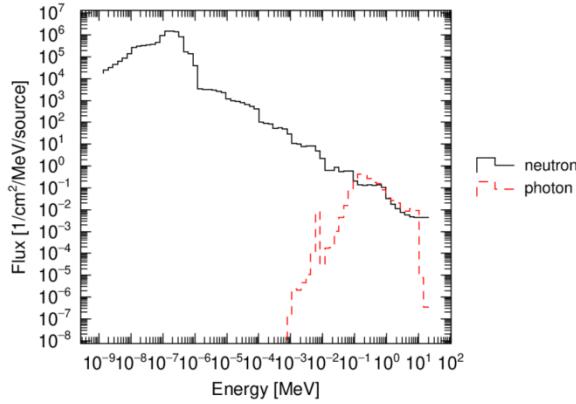


Fig. 8. Neutron-Gamma spectra simulated at the collimator.

Closed shutter

When the shutter is closed (Figure 9), the neutron flux is significantly reduced compared to the collimator alone, illustrating the effect of the shutter that blocks a large portion of the neutrons. For thermal neutrons (energy < 0.1 MeV), the flux is considerably reduced, reaching around 10^5 [1/cm²/MeV/source], demonstrating the shutter's efficiency in blocking these neutrons. For fast neutrons (energy > 1 MeV), the flux becomes extremely low, barely reaching 10^{-4} [1/cm²/MeV/source], indicating a near elimination of fast neutrons by the closed shutter. For photons, the photon flux is also reduced, following a similar trend. It starts to increase significantly around 10^{-3} MeV, with a peak photon flux around 10^{-1} MeV, reaching a flux of approximately 1.4937×10^{-2} [1/cm²/MeV/source].

Beyond 1 MeV, the flux decreases slightly, reaching a value of 3.8078×10^{-8} [1/cm²/MeV/source] at 10 MeV.

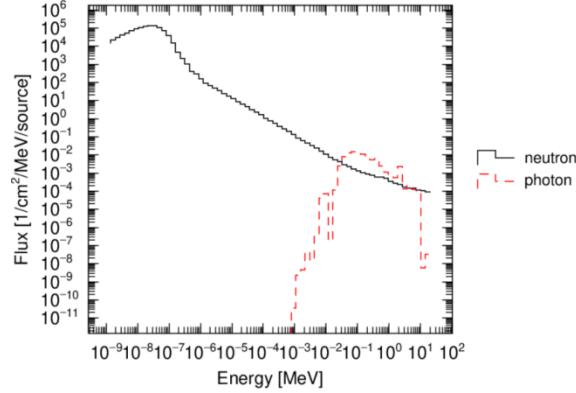


Fig. 9. Neutron- Gamma spectra simulated at closed shutter.

Open shutter

When the shutter is open (Fig. 10), the neutron flux for thermal neutrons (energy < 0.1 MeV) is about 10^6 [1/cm²/MeV/source] similar to that of the collimator alone, indicating that opening the shutter has little impact on the propagation of thermal neutrons, which remain dominant in this energy range. For fast neutrons (energy > 1 MeV), the flux is around 10^{-3} [1/cm²/MeV/source] which is also comparable to that observed with the collimator alone, suggesting that opening the shutter does not significantly affect the attenuation of fast neutrons.

Regarding photons, their flux begins to increase significantly around 10^{-3} MeV, reaching a peak around 10^{-1} MeV, with a flux of approximately 5.9599×10^{-2} [1/cm²/MeV/source]. Beyond 1 MeV, the flux decreases slightly, reaching a value of 1.0039×10^{-6} [1/cm²/MeV/source] at 10 MeV.

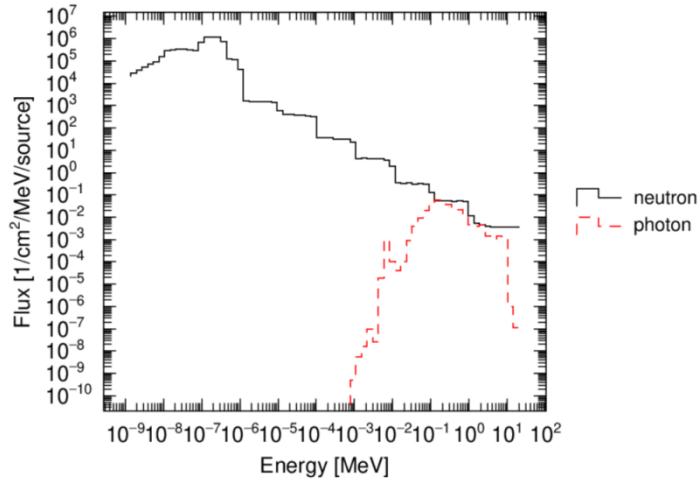


Fig. 10. Neutron -Gamma spectra simulated at open shutter.

The joint analysis of neutron and photon spectra reveals the distinct behavior of the PGAA system depending on the configuration (steel collimator only, shutter closed, or shutter open). The setup demonstrates a clear ability to modulate neutron transport based on energy, with a particularly strong attenuation observed for fast neutrons when any obstacle is introduced in their trajectory. The simulated neutron and photon spectra obtained with the PHITS code are consistent with those generated using the MCNP code [4], [11]. In the collimator configuration, the spectrum exhibits the expected profile of an effective moderation system optimized for the transmission of thermal neutrons. The sharp decrease in fast neutron flux indicates good control over the undesirable high-energy component, which is critical for low-energy-sensitive analyses such as the PGAA technique.

The introduction of the closed shutter acts as an energy filter, significantly reducing the contribution of thermal neutrons and almost entirely eliminating that of fast neutrons. This configuration confirms the key role of the shutter in radiological and nuclear safety, as well as in beam confinement during non-irradiation periods. When the shutter is open, a behavior very similar to that observed with the collimator alone is seen, indicating that the system is designed to retract without compromising the quality of the useful beam. This demonstrates the good stability of the PGAA transmission system during nominal operation.

Regarding photons, their emergence from $\sim 10^{-3}$ MeV and the peak around 10^{-1} MeV confirm their secondary nature, mainly produced by neutron-matter interactions. Their flux varies depending on the configuration but remains within expected orders of magnitude, suggesting consistency with capture phenomena, inelastic scattering, and subsequent gamma production.

Overall, the results reflect a well calibrated system where the performance of each component (steel collimator, open shutter and closed shutter) aligns with its theoretical function, ensuring effective control of the energy spectrum and targeted attenuation according to experimental and nuclear radiological safety requirements.

4. CONCLUSIONS

In conclusion, the results from the simulations conducted using the PHITS code provided a detailed analysis of the flux distribution and energy spectra of neutrons and gamma rays in the PGAA system. The results confirmed the effectiveness of the materials used, particularly the Al_2O_3 sapphire filter as well as the steel and HDPE rings, in attenuating parasitic radiation and optimizing neutron beam concentration at the center of the PGAA system. When the beam shutter is closed, the drop in neutron and photon flux especially fast neutrons is significant, validating the efficiency of this setup in blocking undesirable radiation. Conversely, when the beam shutter is open, the particle flux increases, ensuring the full transmission of the main beam through the steel collimator. These results highlight the importance of the beam shutter in adjusting beam propagation according to experimental requirements, thus ensuring precise radiation control and greater efficiency in PGAA applications. In summary, this article validates the design of the PGAA system and opens promising prospects for optimizing future experiments.

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