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# DETERMINING DIFFERENT PLANT LEAVES' FRACTAL DIMENSIONS: A NEW APPROACH TO TAXONOMICAL STUDY OF PLANTS

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*Key words:* Fractal dimension, Box-Counting Method, Density-density correlation

## Abstract

Fractal dimensions of leaves from *Cercis canadensis* L., *Robinia pseudoacacia* L., *Amelanchier arborea* (F.Michx.) Fernald, *Prunus persica* (L.) Batsch, *Quercus alba* L., *Carpinus caroliniana* Walter, *Ficus carica* L., *Morus rubra* L., *Platanus orientalis* L., and *Ulmus rubra* Muhl. were calculated. The values were then confirmed and compared by those obtained from box-counting method and the exponent values of density-density correlation function (first time in the literature). It is now proposed for the first time that there is a relationship between a fractal dimension of the leaf and a surface density of the image and was concluded that together with other measures, the fractal dimensions with surface density function could be used as a new approach to taxonomical study of plants.

## Introduction

Fractals are complex geometric figures made up of small scale and large-scale structures that resemble one another. Generally, there are two types: geometric (regular) and non-geometric (irregular). A geometric fractal consists of large and small structures that resemble precise duplication of each other. In irregular fractals, there are also large and small structures, but they do not resemble to each other. Instead, the structures are geometrically related. Irregular fractals have many patterns in nature. Fractal geometry offers simple descriptions of some elaborate fern shapes (Campbell 1996). Structures that grow by continually repeating simple growth rules are suitable model for L-systems and fractal analysis (Barnsley *et al.* 1986, Fitter and Stickland 1992, Prusinkiewicz *et al.* 1996). Talking about fractals we usually think of the fractal dimension. Topology is a branch of mathematics which has essentially been developed in 20th century. It deals with questions of form and shape from a qualitative point of view. Two of its basic notions are "dimension" and "homeomorphism". Topology deals with the way shapes can be pulled and distorted in a space that behaves like rubber (Peitgen *et al.* 2004).

At the turn of the last century it was one of the major problems in mathematics to determine what dimension means and which properties it has. And since then mathematicians have come up with some ten different notions of dimension: topological dimension, Hausdorff dimension, fractal dimension, self-similarity dimension, box-counting dimension, capacity dimension, information dimension, Euclidean dimension, and more. Some of them, however, make sense in certain situations, but not at all in others, where alternative definitions are more helpful. Sometimes they all make sense and are the same. In general, many researches mainly focus on only three of these dimensions, namely self-similarity dimension, compass dimension (also called divider dimension) and box-counting dimension. And also, of these three notions of dimension the box-counting dimension has the most applications in science. The reason for its dominance lies in the easy and

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automatic computability by machine. It is straightforward to count boxes and to maintain statistics allowing dimension calculation. The program can be carried out for shapes with and without self-similarity (Falconer 1990).

Fractal geometry has found many applications in the sciences in the last few decades. Examples include the classification and analysis of dynamical systems, modeling of diffusion processes in statistical mechanics, classifying surface roughness, analyzing crack propagation in solids, and studying the spread of forest fires and infectious diseases, to list a very few. Study done by Bayırlı (2005) calculated the fractal dimension of manganese dendrites that formed on surface of magnesite ore. These kinds of studies and applications have been becoming very popular in biological science and ecology as well (Williamson and Lawton 1991). Borkowski (1999) analyzed more than 300 leaves from 10 tree species and reported several classical biometric descriptors as well as 16 fractal dimension features on digitized leaf silhouettes. It has been point out that properly defined fractal dimension based features may be used to discriminate between species, especially when used together with other measures. Because of this, they can be utilized in computer identification systems and for taxonomical purposes (Borkowski 1999).

Pita *et al.* (2002) measured fractal dimension of *Asparagus plumosus* that is cultivated as an ornamental plant. The plant displays self-similarity, extraordinary in at least two different scaling. They demonstrated their results by analyzing this plant via the box counting method for three different scalings.

Vleck and Cheung (1986) measured the fractal dimension of leaf edges in a number of species. Although D, indicates the topological dimension, was found to be highly variable in some species (e.g. oaks), they felt that D might have potential as a taxonomic character. The fractal dimension of root systems was examined by Tatsumi *et al.* (1989) using the box-counting method. They found fractal dimensions in the range 1.46 - 1.6 for mature crop plants.

Fitter and Stickland (1992) demonstrated that the fractal dimension of root systems increases over time (to a maximum D of approximately 1.35), and varies between species. Corbit and Garbary (1995) found no differences in the fractal dimension of three algal species, though D increased with both developmental stage and frond structural complexity. Mancuso (2001) calculated fractal dimension (which varied between 1.204 and 1.499) of grapevine (*Vitis* sp.) leaves belonging to different genotypes.

The multi-scale function of the Murkowski fractal dimension, which is another analyzing method, was applied to digital images of leaves to generate complexity measure of their internal and external form. According to Rodrigo *et al.* (2005) if correctly differentiating among species, this method might be very useful. Keller *et al.* (1989) described a modified box-counting texture analysis technique based on the probability density function. They characterized simulated (Brodatz) textures in terms of fractal dimension and lacunarity. An alternative box-counting method was proposed by Sarkar and Chaudhuri (1992). In their method, each x, y coordinate has associated third dimension (z-coordinate) representing pixel intensity (e.g. gray shade). The box count is determined as the number of cells in a column intercepted by the surface. Using simulated textures, they found that their method was more computationally efficient than that proposed by Keller *et al*. (1989) and De Cola (1993) described a hierarchical grid method for the analysis of surface texture in remotely sensed images. It was found that fractal dimension varied with scale, implying multi-fractal behavior.

Classical taxonomy usually uses morphological and anatomical characteristics of the plants. In addition to these, in today's taxonomy, characters from different sources, like palynology, embryology, cytology, chemistry are also used. Although some recent studies, as indicated above, showed that features based on fractal dimension can be used for taxonomic purpose and they can be a taxonomic character of the plants, Halley *et al.* (2004) reviewed the value of fractal methods, particular for applications to spatial ecology, and outlined potential pitfalls.

They stressed the limitations and the strengths of fractal models. They argued that no ecological pattern can be truly fractal, however fractal methods may nonetheless provide the most efficient tool available for describing and predicting ecological patterns at multiple scales.

The main purpose of this study was to determine the fractal dimensions of ten different plant leaves by using box-counting method. Then, the values obtain from this method were verified and compared by calculating exponent values of density-density function. Lastly, relationships between fractal dimensions and cell (here pixel) density of the image surface were investigated.

## Materials and Methods

To determine fractal dimensions leaves of *Cercis canadensis* L., *Robinia pseudoacacia* L., *Amelanchier arborea* (F. Michx.) Fernald, *Prunus persica* (L.) Batsch, *Quercus alba* L., *Carpinus caroliniana* Walter, *Ficus carica* L., *Morus rubra* L., *Platanus orientalis* L., and *Ulmus rubra* Muhl.) belonging to seven different families were selected (Davis 1965-1985, Stearn 1973, Brummitt and Powell 1992, Akçiçek *et al.* 2005). The selected leaf samples were scanned and then transferred to computer to make necessary analyses (Fig. 1).

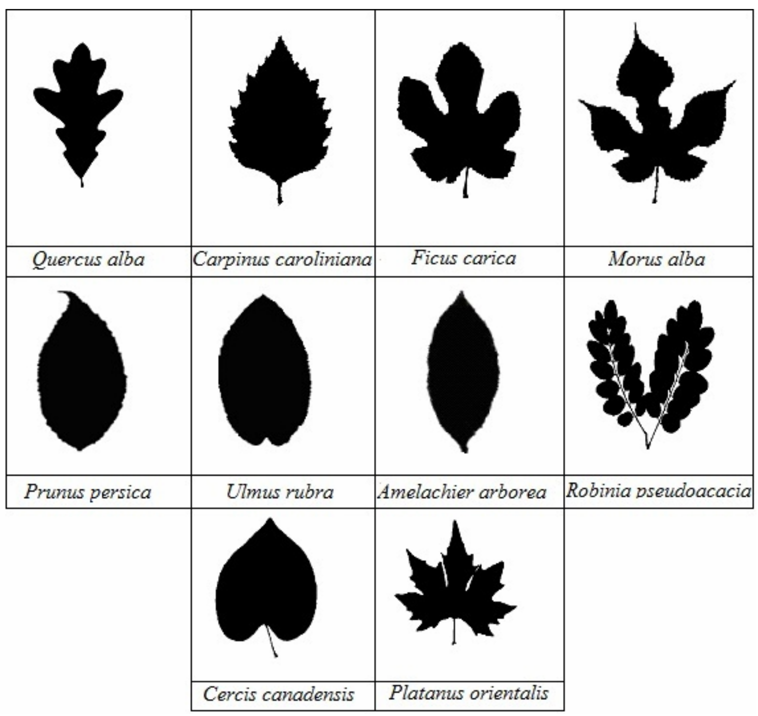


Fig. 1. The real images of ten different trees' leaves obtained by a scanner.

All the images were obtained directly on a scanner (640 × 480 resolution, bit map images), and as the leaves are objects immersed in a two dimensional space. The digital scanning was made in black ("on"-1) and white ("off"-0), and in real scale. The side size of square grid varied from 1 to 300 pixels, by steps of one pixel (an example of a grid of boxes with a side-length of image

(Fig. 2). Changes that might be occurred on leaf surfaces are ignored due to the natural, climatic or physiologic impacts. The leaves with smooth surfaces are especially preferred. As our images all have well defined borders, there is no need to analyze the contour threshold (Prasad and Sreenivasan 1990).

The software program used in this study was especially developed for calculating fractal dimensions of any scanned images. This program was used previously to calculate the fractal dimension of manganese dendrites that formed on surface of magnesite ore (Bayırlı 2005).

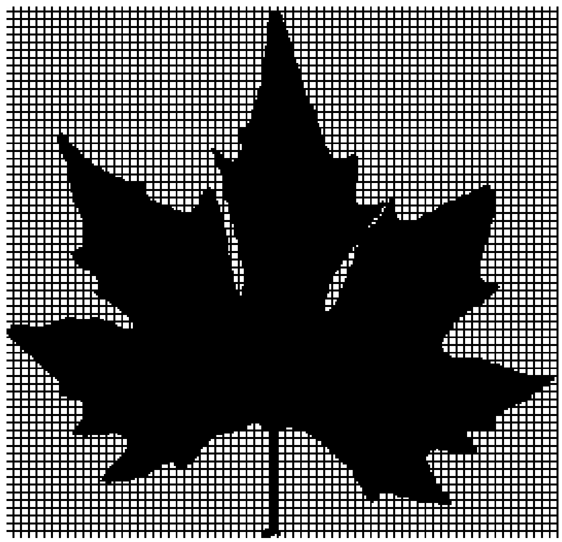

Fig. 2. A grid box-image with side-length of 285 pixels - an example of *Platanus orientalis*.

To calculate fractal dimension values of ten different plant leaves, there are many methods are used; however, for the research purpose the grid (box-counting) method was used. And then, the results were compared and verified by using another method, density-density correlation functions. To find out the relationships fractal dimensions and cell density of the image surface were investigated. Fractal dimensions and cell density of the image surface were investigated to find out their relationship. Some detail of these methods have been given below.

*Box-counting (grid) method:* The box-counting dimension, another special form of Mandelbrot's fractal dimension, proposes a systematic measurement, which applies to any structure in the plane and can be readily adapted for structures in space. This procedure can be used to measure the fractal dimension of a curve (1989). The box-counting method can also been used to determine the fractal dimension of pixel images (Milne 1992, Virkkala 1993). Longley and Batty (1989) noted that the box-counting method may be less suited to the task of hugging the more intricate details of the base curve but, because of its low computer processing requirements; it is recommended as a method suitable for yielding a first approximation to the fractal dimension. Pruess (1995) demonstrated that the limited resolution of most data renders the estimation of D sensitive to the range of box lengths used.

An image of leaf is considered in which states of interest are coded either 'on' (1) or 'off' (0). To determine the fractal dimension of the 'on' pixels, divide the image into coarser scales of pixel resolution ('windows or lattice') is divided and count the number of lattice (N) occupied by a least one 'on' pixel (length of ε) is counted. The log-log plot (resolution scale vs. number of windows occupied) is used to determine the fractal dimension (D = slope). The fractal dimension of any object is given by the following equation:

$$D = \lim_{\varepsilon \to 0} \frac{\log N(\varepsilon)}{\log(1/\varepsilon)} \qquad (1)$$

De Cola (1993), described a related method, based on hierarchical grouping of adjacent pixels, for determining the fractal dimension of spatial auto-correlation.

*Density-density correlation function:* The density-density correlation function C (ε) of leaves, an approximation to the average correlation function gathered from a similar cluster group, can be described as:

$$C(\varepsilon) = \frac{1}{N} \sum_{\vec{\varepsilon}'} \rho(\vec{\varepsilon}')\rho(\vec{\varepsilon}' + \vec{\varepsilon}) \qquad (2)$$

In the given equation, where N is the total cell (here-lattice) number in the cluster and ρ (ε) is the cell density. It is supposed that this function depends on ε, distance between cells; this assumption is only valid if the ε value is smaller than dimensions of the cluster. The correlation function of the fractal cluster is given by the following equation:

$$C(\varepsilon) = K\varepsilon^{-\alpha} \qquad (3)$$

where K is a constant number and □ is an exponent value of correlation function. The value of □ is computed by fitting log C (ε) vs. log (ε). A slope value of the logarithm is relevant in the fit interval with 96 - 99% correctness. Therefore; there is relationship between exponent value of the density-density correlation function and the value of fractal dimension.

When the off-square lattice is filled with objects, the lattice dimension of the cluster is given by d (which indicates Euclidian dimension). The Euclidian dimension has to be exact values such as 1, 2, 3.... There is also relationship between the value of fractal dimension and Euclidian dimension. This relation is given by the following equation:

$$D = d - \alpha \qquad (4)$$

where d is constant for the Euclidian dimension-for 2D of any object, its value is 2-; D is the fractal dimension. The slope quantity of log N (ε) vs. log (ε) plot gives the fractal dimension value of the species. The equations 1 and 4 are two different approaches; however, their results must be equal for finding the fractal dimension of objects.

*Cell (Pixel) density of surface (ρ):* Cell density of the image surface (ρ) can be defined as a total number of cell (N) is divided by square of length (ε) of the cell. To show a relationship between fractal dimension and cell (here pixel) density of the image surface, the following equation was proposed:

$$\langle \rho \rangle = \beta D + C \qquad (5)$$

where β is the slope of the fitted value between the cell density and fractal dimension value, *D*; and C is universal constant number. This equation gives different result for each plant leaf; therefore, its results can be useful for categorizing plant leaves according to their cell density of surfaces.

**Results and Discussion**

The characteristics of leaves by means of a total number of counted cells, fit intervals, critic exponent values, fractal dimensions and cell density of surface areas are given in Table 1.

It can be seen in Table 1 that the exponent value of density-density correlation function changes between 0.068 and 0.473 for the leaf of *Carpinus caroliniana* and *Moris ruba* L., respectively. The fractal dimensions vary between 1.682 and 1.816 for box counting method. The cell density of surface area of leaves vary between 0.330 and 0.822.

**Table 1. The characteristics of leaves by means of a total number of counted cells, fit intervals, critic exponent values, fractal dimensions and cell density of surface areas.**

| Image name | Number of cells | Fit interval | Alpha (α) | D (D=d-2)[a] | ρ | D (Box counting)[b] |
|---|---|---|---|---|---|---|
| *Quercus alba* L. | 11871 | 1 - 31 | 0.163 ± 0.012 | 1.733 ± 0.010 | 0.822 | 1.711 |
| *Carpinus caroliniana* Walter | 26544 | 3 - 16 | 0.068 ± 0.003 | 1.927 ± 0.011 | 0.612 | 1.816 |
| *Ficus carica* L. | 18662 | 4 - 27 | 0.176 ± 0.008 | 1.837 ± 0.009 | 0.430 | 1.749 |
| *Morus rubra* L. | 15005 | 2 - 24 | 0.473 ± 0.019 | 1.757 ± 0.009 | 0.330 | 1.693 |
| *Prunus persica* (L.) Batsch | 57446 | 2 - 28 | 0.125 ± 0.005 | 1.864 ± 0.005 | 0.484 | 1.746 |
| *Ulmus rubra* Muhl. | 23788 | 1 - 27 | 0.095 ± 0.006 | 1.910 ± 0.015 | 0.587 | 1.708 |
| *Amelanchier arborea* (F.Michx.) Fernald | 28438 | 4 - 15 | 0.070 ± 0.003 | 1.980 ± 0.015 | 0.701 | 1.749 |
| *Robinia pseudoacacia* L. | 16268 | 3 - 16 | 0.091 ± 0.006 | 1.922 ± 0.015 | 0.565 | 1.682 |
| *Cercis canadensis* L. | 24309 | 3 - 23 | 0.080 ± 0.004 | 1.811 ± 0.009 | 0.509 | 1.806 |
| *Platanus orientalis* L. | 32206 | 3 - 19 | 0.103 ± 0.005 | 1.757 ± 0.009 | 0.389 | 1.692 |

A typical fractal dimension plot logN(ε), versus log(1/ε) and a plot of density-density correlation function of leaves vs. length of lattice logC(ε) vs. log(ε) are shown in Fig. 3a and b, respectively.

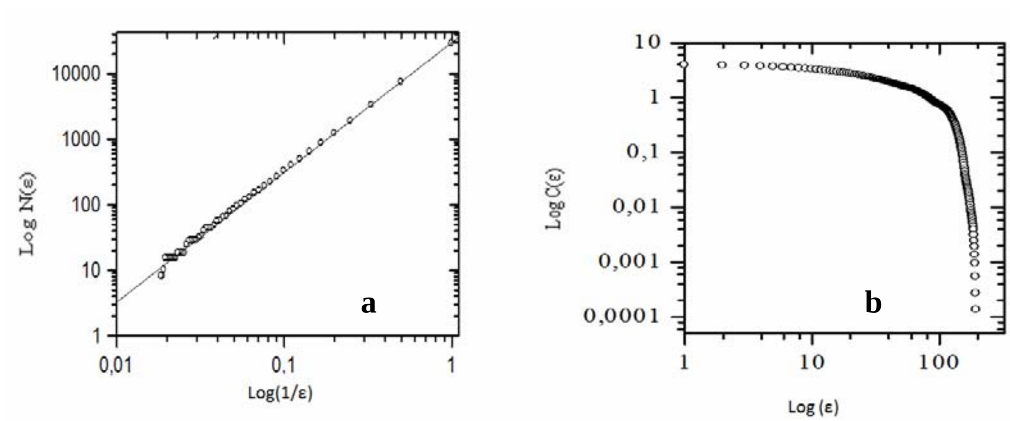


Fig. 3a. A typical fractal dimension plot (logN(ε), versus log( 1/ε). b: A plot of density-density correlation function of leaves vs. length of lattice (LogC(ε) vs Log ε).

It can be seen in Fig. 3a that the slope of this plot gives fractal dimension value for box counting method. Fig. 3b shows the relation between density-density correlation function of leaves and logarithmic value of ε. A relationship between fractal dimension (D) and cell surface density (ρ) of the images are given in Fig. 4.

As shown in Fig. 4, there is linear correlation between fractal dimension of leaves and cell density of image surface.

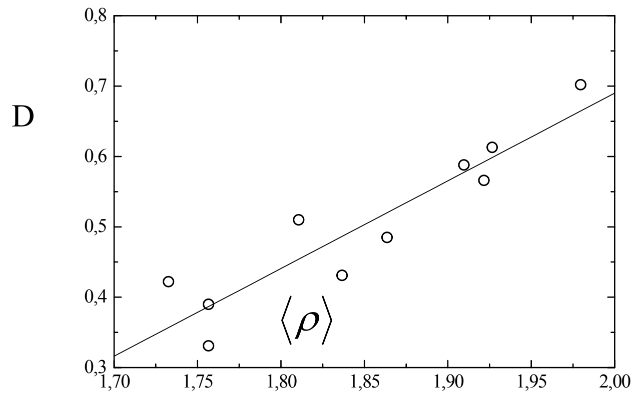


Fig. 4. A relationship between fractal dimension (D) and cell density (ρ) of the image surface.

By using box-counting (grid) method, it is found that simple leaves of *Carpinus caroliniana*, *Prunus persica*, *Ulmus rubra*, *Amelanchier arborea* and *Cercis canadensis* have similar structural geometry. Thus, the fractal dimension values varied between 1.811 and 1.980. The fractal dimension values of *Quercus alba*, *Ficus carica*, *Morus rubra* and *Platanus orientalis,* which they have deeply palmately lobed leaves, varied between 1.733 and 1.837. The fractal dimension value of *Robinia pseudoacacia*, which has compound leaves, was calculated as 1.864.

To verify the results obtained from box-counting method, the second method, density-density correlation function, was applied all plant leaves. It is found that the exponent values of the density-density correlation function for *Carpinus caroliniana*, *Prunus persica*, *Ulmus rubra*, *Amelanchier arborea* and *Cercis canadensis* varied between 0.068 and 0.095; for *Quercus alba*, *Ficus carica*, *Morus rubra* and *Platanus orientalis* varied between 0.103 and 0.473; and for *Robinia pseudoacacia* is calculated as 0.125. When these critical exponent values were plugged into equation 4, we obtained fractal dimensions of all species (as indicated in Table 1). In this way we proposed, the first time in literature, that fractal dimensions of leaves can be calculated for this way to double checking purposes or to use by itself alone. When we compared the results obtained by two methods, almost all fractal values are found very close to each other except *Robinia pseudoacacia* L.

Because of isotropic and anisotropic influences during their developments, natural leaf geometry faces many unexpected conditions. Therefore, it is not always possible to see the same structure in all species of leaves. It is impossible to find exactly same leaves in any species. Some fluctuations in the fractal dimension occur according to physiology and climatic changes and due to the fact that geometric structure of the leaves changes within a period. Consequently, it is not totally possible to discriminate tree leaves in genus level by looking only their fractal dimension values. Besides, some differences in the leaf shapes could produce different value in fractal dimension (for instance, according to the amounts of separation in the edges; leaves of *Amelanchier arborea* had a higher fractal value than the other leaves stated in this study). For this reason, the new approach is needed to describe and categorize the structure of leaves, proposed also first time in this study and given in equation 5, which is called cell density of leaf surface, ρ.

In equation 5, it was found that the slope, β, equals 1.247 ± 0.183 and the universal constant value of C, equals 1.803 ± 0.338. Therefore, the cell density of leaf surface (ρ) varied for all species between 0.330 and 0.822. The geometric difference of the leaves influences the cell density of leaf because the lattice dimensions are taken as a reference to each edge of leaf. In this study, as *Morus rubra* has deeply divided leaves, and has the greatest in terms of off-lattice (yet, it aggregates a fewer cells); therefore, the value of cell density of leaf surface is found the smallest. On the contrary, *Quercus alba* has slightly divided leaves and has the smallest with respect to off-lattice; yet, the value of cell density of its surface is found the greatest over all.

Though Halley *et al.* (2004) discussed that no ecological pattern can be truly fractal, however fractal methods may nonetheless provide the most efficient tool available for describing and predicting ecological patterns at multiple scales, recognition of the fractal geometry of nature has important implications to biology, as evidenced by the several examples presented here. Zeide and Gresham (1991) described as 'self-evident' the fractal nature of biological structures and systems. One of the enormous challenges facing biologists lies in translating these self-evident concepts into comprehensive models of the patterns and processes observed in natural world.

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